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Technical Notes

Heat Transfer of Magnetohydrodynamic Viscous Nanofluids over an Isothermal Stretching Sheet

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Nomenclature

B	=	magnetic field strength
c	=	constant
D_B	=	Brownian diffusion coefficient
D_T	=	thermophoretic diffusion coefficient
k	=	thermal conductivity
Le	=	Lewis number
M	=	magnetic parameter
Nb	=	Brownian motion parameter
Nt	=	thermophoresis parameter
Nu	=	Nusselt number
Nur	=	reduced Nusselt number
Pr	=	Prandtl number
p	=	pressure
$(\rho c)_f$	=	heat capacity of the fluid
$(\rho c)_p$	=	effective heat capacity of the nanoparticle material
q_m	=	wall mass flux
q_w	=	wall heat flux
Re_x	=	local Reynolds number
Shr	=	reduced Sherwood number
Sh_x	=	local Sherwood number
T	=	fluid temperature
T_w	=	temperature at the stretching sheet
T_∞	=	ambient temperature
u, v	=	velocity components along x and y axis
u_w	=	velocity of the stretching sheet
x, y	=	Cartesian coordinates (x axis is aligned along the stretching surface and y axis is normal to it)
α	=	thermal diffusivity
β	=	dimensionless nanoparticle volume fraction
η	=	similarity variable
θ	=	dimensionless temperature
ρ_f	=	fluid density
ρ_p	=	nanoparticle mass density
σ	=	electrical conductivity of the fluid
τ	=	parameter defined by ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid. $\tau = (\rho c)_p/(\rho c)_f$
φ	=	nanoparticle volume fraction
φ_∞	=	ambient nanoparticle volume fraction
φ_w	=	nanoparticle volume fraction at the stretching sheet
ψ	=	stream function

I. Introduction

THE hydromagnetic flow and heat transfer of a viscous fluid passover continuously moving surface has many practical applications, especially in the field of metallurgy and chemical engineering, such as extrusion of polymer plastic sheets, cooling of metallic plate, drawing of paper films, glass blowing, paper production, and magnetohydrodynamic (MHD) pumps [1].

Magnetic nanofluids are unique materials that have both the liquid and magnetic properties. Many of the physical properties of these nanofluids can be tuned up by varying the magnetic field. The magnetic nanofluids are easy to manipulate with an external magnetic field; therefore, they have been used for variety of studies [2].

Nanofluid is described as a fluid in which solid nanoparticles with the length scales of 1–100 nm are suspended in conventional heat transfer basic fluid. It has been recognized that the addition of highly conductive particles can significantly increase the thermal conductivity of basic pure fluids. Buongiorno [3] developed an analytical model for convective transport in nanofluids in which Brownian motion and thermophoresis effect are taken into account. Recently, the Boungiorno's model [3] has been used by Kuznetsov and Nield [4]. They studied the influence of nanoparticles on natural convection boundary-layer flow past a vertical plate by considering the effects of Brownian motion and thermophoresis parameters.

Very recently, Khan and Pop [5] analyzed flow and heat transfer of nanofluids over a stretching sheet by considering dynamic effects of nanoparticles including Brownian motion and thermophoresis. Following that, Makinde and Aziz [6] applied convective boundary condition on the sheet while Rana and Bhargava [7] considered a nonlinear velocity for the stretching sheet. Hassani et al. [8] applied the Homotopy analyses method and analytically analyzed model of Khan and Pop [5]. Noghrehabadi et al. [9] considered the development of partial slip effects on the boundary-layer heat transfer of nanofluids in the presence of Brownian motion and thermophoresis forces over an isothermal stretching sheet.

The present study aims to examine the combined effects of Brownian motion, thermophoresis, and magnetic field on the steady boundary-layer flow and heat transfer of nanofluids over a linear isothermal stretching sheet.

II. Mathematical Model

Consider a two-dimensional viscous flow of a nanofluid over a stretching surface in a quiescent fluid, in which the flow is incompressible and steady-state. A uniform transverse magnetic field with strength B is applied in the vertical direction. It is assumed that the induced magnetic field is negligible in comparison with applied magnetic field (as the magnetic Reynolds number is small), and the external electric field and the electric field due to the polarization of charges are negligible [10,11].

The stretching velocity of the sheet is linear; hence, it can be represented as $U = c.x$ where c is a constant, x is the coordinate measured along the stretching sheet, and y is the coordinate measured normal to the stretching sheet. The coordinate system and the scheme of the problem are shown in Fig. 1. Although there are three distinct boundary layers over the stretching sheet, only a single boundary layer is shown to avoid congestion. The boundary layers over the sheet are the hydrodynamic (velocity) boundary layer, the thermal

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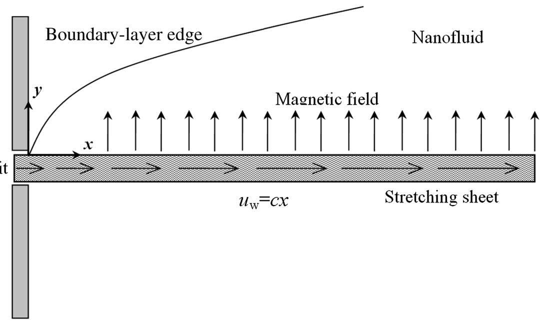

Fig. 1 Boundary-layer configuration.

boundary layer (temperature), and the concentration boundary layer (volume fraction of nanoparticles).

The flow takes place at $y = 0$. It is assumed that the wall temperature takes the constant value of T_w and the fraction of nanoparticles takes the constant value of φ_w at the stretching surface. When y tends to infinity, the ambient values of temperature and nanoparticle fraction attained to the constant value of T_∞ and φ_∞, respectively. The variables φ and T denote a fraction of nanoparticles and temperature of flow, respectively. Under the previous assumptions, in a Cartesian coordinates system of x and y the basic steady conservation of mass, momentum, thermal energy, and nanoparticle equations for nanofluids can be obtained as follows [4,11]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x} + \nu\nabla^2 u - \frac{\sigma B^2}{\rho_f}u \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p}{\partial y} + \nu\nabla^2 v \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\nabla^2 T + \tau\left\{D_B\nabla\varphi\cdot\nabla T + \frac{D_T}{T_\infty}\nabla T\cdot\nabla T\right\} \tag{4}$$

$$u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} = D_B\nabla^2\varphi + \left(\frac{D_T}{T_\infty}\right)\nabla^2 T \tag{5}$$

The boundary conditions at the wall (i.e., $y = 0$) are defined as

$$v = 0, \qquad u = U_w(x), \qquad T = T_w, \qquad \phi = \phi_W, \quad \text{at } y = 0 \tag{6}$$

The boundary conditions at the far field (i.e., $y \to \infty$) are defined as

$$v = u = 0, \qquad T = T_\infty, \qquad \phi = \phi_\infty, \quad \text{as } y \to \infty \tag{7}$$

The parameters of the above equations are introduced in the Nomenclature.

To attain a similarity solution of Eqs. (1–5) subject to Eqs. (6) and (7), the stream function and dimensionless variables can be posited in the following form:

$$\psi = (c\nu)^{1/2}xf(\eta), \qquad \eta = \left(\frac{c}{\nu}\right)^{1/2}y$$
$$\theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty}, \qquad \beta(\eta) = \frac{\varphi - \varphi_\infty}{\varphi_W - \varphi_\infty} \tag{8}$$

The stream function ψ is defined by $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$, so that the continuity equation will be satisfied identically. By applying the boundary-layer assumptions and the similarity transforms on the remaining governing Eqs. (2–5), the following similarity equations are obtained:

$$f''' + ff'' - f'^2 - Mf' = 0 \tag{9}$$

$$\frac{1}{pr}\theta'' + f\theta' + Nb\beta'\theta' + Nt\theta'^2 = 0 \tag{10}$$

$$\beta'' + \frac{Nt}{Nb}\theta'' + Lef\beta' = 0 \tag{11}$$

Subject to the following boundary conditions:

$$\text{At } \eta = 0\text{: } f = 0, \qquad f' = 1, \qquad \theta = 1, \qquad \beta = 1 \tag{12}$$

$$\text{At } \eta \to \infty\text{: } f' = 0, \qquad \theta = 0, \qquad \beta = 0 \tag{13}$$

where primes denote differentiation with respect to η. The parameters of Pr, Le, M, Nb, and Nt are defined by

$$Pr = \frac{\nu}{\alpha}, \qquad Le = \frac{\nu}{D_B}, \qquad M = \frac{\sigma B^2}{(\rho c)_f}$$
$$Nb = \frac{(\rho c)_P D_B(\phi_W - \phi_\infty)}{(\rho c)_f \nu}, \qquad Nt = \frac{(\rho c)_P D_T(T_W - T_\infty)}{(\rho c)_f \nu T_\infty} \tag{14}$$

The quantities of the local Nusselt number Nu and Sherwood number Sh as important quantities in heat transfer are

$$Nu = \frac{xq_W}{k(T_W - T_\infty)}, \qquad Sh = \frac{xq_m}{D_B(\varphi_W - \varphi_\infty)} \tag{15}$$

where q_w is the heat flux and q_m is the wall mass flux. Using defined similarity transforms in Eq. (8), one can obtain

$$Re_x^{-1/2}Nu = -\theta'(0), \qquad Re_x^{-1/2}Sh = -\beta'(0) \tag{16}$$

Here, $Re_x = u_w(x)x/\nu$ is the local Reynolds number based on the stretching velocity [i.e., $u_w(x)$]. Kuznetsov and Nield [4] referred the value of $Re_x^{-1/2}Nu$ as the reduced Nusselt number Nur, and the value of $Re_x^{-1/2}Sh$ as the reduced Sherwood number Shr.

III. Results and Discussion

The set of ordinary differential Eqs. (9–11) are solved numerically for the various range of the magnet parameter and for selected values of Pr, Le, Nb, and Nt. Numerical results were obtained using a finite difference method basis on collocation points and Newton's method [12,13]. The accuracy of the solution was controlled less than 10^{-6} with an adaptive mesh scheme.

As a test of the accuracy of the solution, the values of Nur are compared with those reported by Hamad [10], Khan and Pop [5], Hassani et al. [8], Gorla and Sidawi [14], and Wang [15] in Table 1 when the effects of thermophoresis, Brownian motion, and magnetic field are neglected.

For a typical case with $Pr = 10$, $Le = 10$, and selected combination of Nt and Nb, the dependent similarity variables $\theta(\eta)$ and $\beta(\eta)$ are plotted for a variation of the magnetic parameter of M in Figs. 2 and 3, respectively. In these figures the zero value of the magnetic parameter simulates the stretching model used in the works of Khan and Pop [5] and Hassani et al. [8]. Both $\theta(\eta)$ and $\beta(\eta)$ tend to zero as the distance increases.

Increase of the magnetic parameter increases the drag force and consequently decreases the magnitude of the temperature and concentration profile as well as thermal and nanoparticle boundary-layer thickness, which are shown in Figs. 2 and 3. Figure 2 shows that the increase of both the thermophoresis and Brownian parameters increases the temperature and thermal boundary-layer thickness. The Brownian motion tends to uniform the concentration of

Table 1 Comparison of results for reduced Nusselt number $-\theta'(0)$

Pr	Hamad [10]	Khan and Pop [5]	Hassani et al. [8]	Gorla and Sidawi [14]	Wang [15]	Present results $Nb = Nt = M = 0$
0.07	0.06556	0.0663	-	0.0656	0.0656	0.065565
0.2	0.16909	0.1691	0.1692	0.1691	0.1691	0.169089
0.7	0.45391	0.4539	0.4582	0.4539	0.4539	0.453916
2	0.91136	0.9113	0.9114	0.9114	0.9114	0.911358
7	1.89540	1.8954	1.8956	1.8905	1.8954	1.895404
20	3.35390	3.3539	3.3539	3.3539	3.3539	3.353917
70	6.46220	6.4621	6.4623	6.4622	6.4622	6.462290

nanoparticles, but, by contrast, the thermophoresis force tends to move nanoparticles from hot to cold areas. These two different effects, i.e., Brownian motion and thermophoresis, on the nanoparticle concentration profiles are shown in Fig. 3. These effects on the concentration profiles are most obvious when the Brownian motion and thermophoresis parameters are comparatively high. Furthermore, the magnetic field strongly influences the areas that are far from the wall when Nb and Nt are comparatively high. However, by decreasing these parameters, the effect of the magnetic field on concentration profiles is also decreased.

The variation of the dimensionless heat transfer rate (i.e., reduced Nusselt number) with respect to the thermophoresis parameter for different values of the magnetic parameter M is shown in Figs. 4 and 5 when $Le = 10$. In these figures, for the case of $M = 0$, neglecting magnetic effects, the obtained profiles of the reduced Nusselt number are compared with results reported by Khan and Pop [5] and Hassani et al. [8]. These figures depict that the reduced Nusselt number is decreased by an increase of the Brownian motion, thermophoresis, or magnetic parameter. Increase of the magnetic parameter decreases the motion of fluid in the boundary layer because of the induced drag force, and thereby, it decreases the reduced Nusselt number. It is interesting that for the water-based nanofluid types ($Pr \approx 10$) the influence of magnetic parameter on the reduced Nusselt number is decreased by increase of the Brownian motion or thermophoresis parameter. By increasing the thermophoresis parameter, the influence of the magnetic parameter on the reduced Nusselt number is increased.

Comparison between Figs. 4 and 5 reveal that the reduced Nusselt number is decreased by the increase of the Prandtl number. Furthermore, the magnetic parameter has a significant effect on the

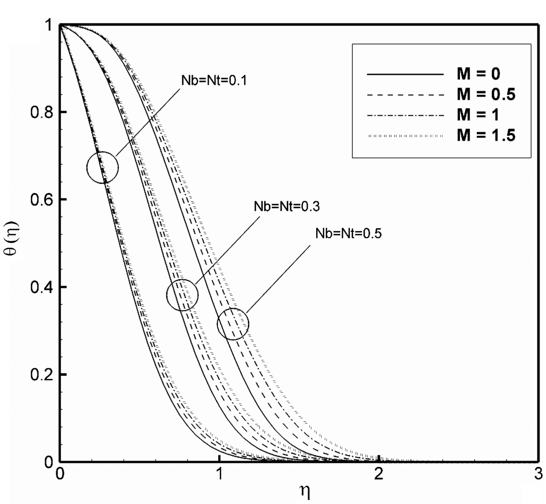

Fig. 2 Temperature profiles for selected values of *M*, *Nb*, and *Nt*.

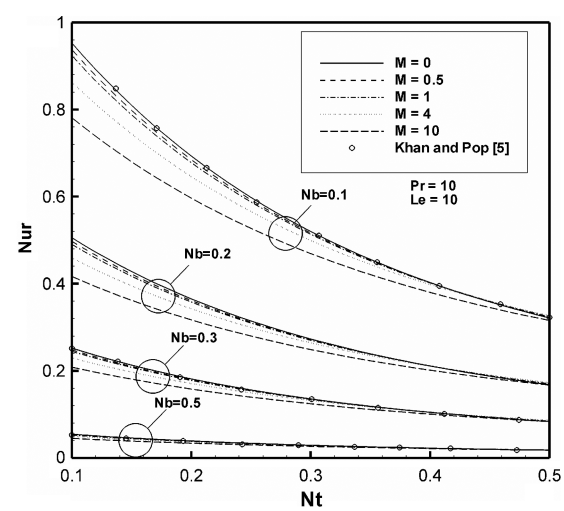

Fig. 4 Variation of *Nur* with *Nt* for different values of *M* and *Nb* when $Pr = Le = 10$.

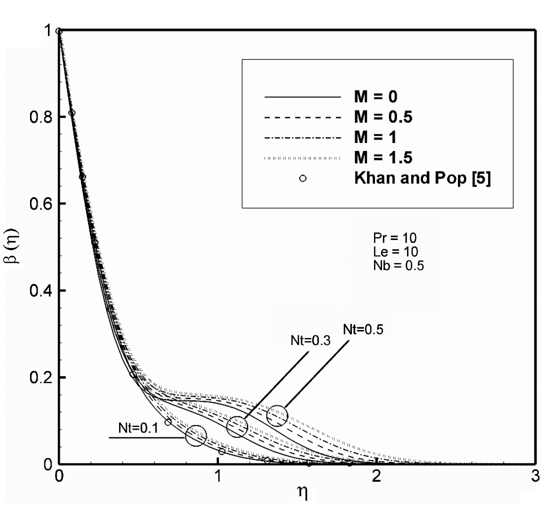

Fig. 3 Profiles of nanoparticle volume fraction for selected values of *M*, *Nb*, and *Nt*.

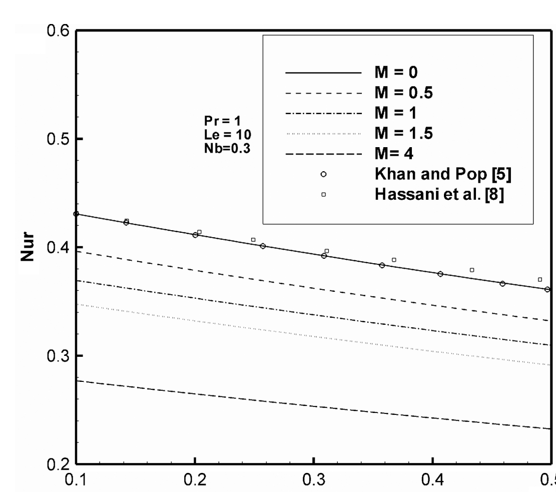

Fig. 5 Variation of *Nur* with *Nt* for different values of *M* when $Pr = 1.0$, $Le = 10$, and $Nb = 0.3$.

reduced Nusselt number even for comparatively large values of the thermophoresis parameter when the Prandtl number is comparatively small.

IV. Conclusions

In the present paper, the effect of the magnetic field on the boundary-layer flow and heat transfer of nanofluids past an isothermal stretching sheet is investigated. The fluid flow boundary-layer governing equations were reduced to a set of nonlinear ordinary differential equations using the similarity transformations and solved numerically for different combinations of nanofluid parameters and magnetic parameter. The findings can be summarized as follows:

1) The thermal boundary-layer thickness is increased by the increase in magnetic parameter.

2) The reduced Nusselt number is a decreasing function of magnetic parameter.

3) For comparatively high values of Brownian motion parameter, the dominant effect on the reduced Nusselt number is the Brownian motion effect while for comparatively small values of Brownian motion parameter, the dominant effects are the magnetic and thermophoresis parameters.

4) The effect of the magnetic parameter on the reduced Nusselt number is decreased by the increase of the thermophoresis parameter or Brownian motion parameter.

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References

[1] Crane, L., "Flow Past a Stretching Plate," *Zeitschrift für Angewandte Mathematik und Physik*, Vol. 21, No. 4, 1970, pp. 645–651. doi:10.1007/BF01587695

[2] Kandasamy, R., Loganathan, P., and Arasu, P. P., "Scaling Group Transformation for MHD Boundary-Layer Flow of a Nanofluid Past a Vertical Stretching Surface in the Presence of Suction/Injection," *Nuclear Engineering and Design*, Vol. 241, No. 6, 2011, pp. 2053–2059. doi:10.1016/j.nucengdes.2011.04.011

[3] Buongiorno, J., "Convective Transport in Nanofluids," *Journal of Heat Transfer*, Vol. 128, No. 3, 2006, pp. 240–245. doi:10.1115/1.2150834.

[4] Kuznetsov, A. V., and Nield, D. A., "Natural Convective Boundary-Layer Flow of a Nanofluid Past a Vertical Plate," *International Journal of Thermal Sciences*, Vol. 49, No. 2, 2010, pp. 243–247. doi:10.1016/j.ijthermalsci.2009.07.015

[5] Khan, W. A., and Pop, I., "Boundary-Layer Flow of a Nanofluid Past a Stretching Sheet," *International Journal of Heat and Mass Transfer*, Vol. 53, Nos. 11–12, 2010, pp. 2477–2483. doi:10.1016/j.ijheatmasstransfer.2010.01.032

[6] Makinde, O. D., and Aziz, A., "Boundary Layer Flow of a Nanofluid Past a Stretching Sheet with a Convective Boundary Condition," *International Journal of Thermal Sciences*, Vol. 50, No. 7, 2011, pp. 1326–1332. doi:10.1016/j.ijthermalsci.2011.02.019

[7] Rana, P., and Bhargava, R., "Flow and Heat Transfer of a Nanofluid over a Nonlinearly Stretching Sheet: A Numerical Study," *Communications in Nonlinear Science and Numerical Simulation*, Vol. 17, No. 1, 2012, pp. 212–226. doi:10.1016/j.cnsns.2011.05.009

[8] Hassani, M., Mohammad Tabar, M., Nemati, H., Domairry, G., and Noori, F., "An Analytical Solution for Boundary Layer Flow of a Nanofluid Past a Stretching Sheet," *International Journal of Thermal Sciences*, Vol. 50, No. 11, 2011, pp. 2256–2263. doi:10.1016/j.ijthermalsci.2011.05.015

[9] Noghrehabadi, A., Pourrajab, R., and Ghalambaz, M., "Effect of Partial Slip Boundary Condition on the Flow and Heat Transfer of Nanofluids Past Stretching Sheet Prescribed Constant Wall Temperature," *International Journal of Thermal Sciences*, Vol. 54, No. 11, 2012, pp. 253–261. doi:10.1016/j.ijthermalsci.2011.11.017

[10] Hamad, M. A. A., "Analytical Solution of Natural Convection Flow of a Nanofluid over a Linearly Stretching Sheet in the Presence of Magnetic Field," *International Communications in Heat and Mass Transfer*, Vol. 38, No. 4, 2011, pp. 487–492. doi:10.1016/j.icheatmasstransfer.2010.12.042

[11] Chamkha, A. J., and Aly, A. M., "MHD Free Convection Flow of a Nanofluid Past a Vertical Plate in the Presence of Heat Generation or Absorption Effects," *Chemical Engineering Communications*, Vol. 198, No. 3, 2010, pp. 425–441. doi:10.1080/00986445.2010.520232

[12] Keller, H. B., "Numerical Solution of Two-Point Boundary Value Problems," *SIAM Regional Conference Series in Applied Mathematics*, J. W. Arrowsmith Ltd., Bristol, England, U.K., 1976.

[13] Russell, R. D., and Shampine, L. F., "A Collocation Method for Boundary Value Problems," *Numerische Mathematik*, Vol. 19, No. 1, 1972, pp. 1–28. doi:10.1007/BF01395926

[14] Gorla, R. S. R., and Sidawi, I., "Free Convection on a Vertical Stretching Surface with Suction and Blowing," *Applied Scientific Research*, Vol. 52, No. 3, 1994, pp. 247–258. doi:10.1007/BF00853952

[15] Wang, C. Y., "Free Convection on a Vertical Stretching Surface," *ZAMM: Journal of Applied Mathematics and Mechanics*, Vol. 69, No. 11, 1989, pp. 418–420. doi:10.1002/zamm.19890691115