Physics-informed neural networks (P INNs): application categories, trends and impact

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Abstract

Purpose – This study aims to explore the evolving field of physics-informed neural networks (PINNs) through an analysis of 996 records retrieved from the Web of Science (WoS) database from 2019 to 2022.

Design/methodology/approach – WoS database was analyzed for PINNs using an inhouse python code. The author's collaborations, most contributing institutes, countries and journals were identified. The trends and application categories were also analyzed.

Findings – The papers were classified into seven key domains: Fluid Dynamics and computational fluid dynamics (CFD); Mechanics and Material Science; Electromagnetism and Wave Propagation; Biomedical Engineering and Biophysics; Quantum Mechanics and Physics; Renewable Energy and Power Systems; and Astrophysics and Cosmology. Fluid Dynamics and CFD emerged as the primary focus, accounting for 69.3% of total publications and witnessing exponential growth from 22 papers in 2019 to 366 in 2022. Mechanics and Material Science followed, with an impressive growth trajectory from 3 to 65 papers within the same period. The study also underscored the rising interest in PINNs across diverse fields such as Biomedical Engineering and Biophysics, and Renewable Energy and Power Systems. Furthermore, the focus of the most active countries within each application category was examined, revealing, for instance, the USA's significant contribution to Fluid Dynamics and CFD with 319 papers and to Mechanics and Material Science with 66 papers.

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Originality/value – This analysis illuminates the rapidly expanding role of PINNs in tackling complex scientific problems and highlights its potential for future research across diverse domains.

Keywords Physics-informed neural networks (PINNs), Application categorizes, Trends and impacts, Geographical distributions, Fluid dynamics and CFD

Paper type General review

1. Introduction

Physics-informed neural networks (PINNs) have emerged as a promising method for incorporating domain-specific knowledge, specifically physics, into the training of neural networks (Raissi *et al.*, 2017). In recent years, the development and utilization of PINNs have exploded, resulting in advancements in disciplines such as computational fluid dynamics (CFD), solid mechanics and heat transfer (Yang *et al.*, 2020; Bai *et al.*, 2023; Cai *et al.*, 2021b). This bibliometric analysis seeks to provide an overview of the PINNs research landscape, highlighting significant contributions, trends and possible future directions.

Raissi *et al.* (2017) introduced PINNs in 2017 as a novel deep-learning technique for solving partial differential equations (PDEs). Incorporating the governing physical equations, typically in the form of PDEs, as constraints during the training of a neural network is the central concept underlying PINNs. This is accomplished by minimizing the composite loss function, which includes data loss and physics-based loss (Raissi *et al.*, 2019). The data loss ensures that neural network predictions are consistent with the available data, whereas the physics-based loss ensures that the underlying physical principles are satisfied. This strategy has been shown to improve the accuracy, generalizability and interpretability of the learned models, as well as reduce the quantity of training data required (Kovachki *et al.*, 2021; Han *et al.*, 2018).

Since the inception of PINNs, the landscape of research has expanded rapidly, with numerous studies investigating various facets of this method. Various architectures and training techniques have been devised by researchers to enhance the effectiveness and performance of PINNs (Bai *et al.*, 2022; Haitsiukevich and Ilin, 2022). Researchers often use a "teaching by example" method, like PDE systems, for scientific tasks involving equations. They provide a model with a question paired with the correct answer, aiming to improve the model's accuracy. However, most common models, which thrive on vast data, can face challenges when data is limited or when strict rules from the equations apply. In response, researchers developed a new technique that ensures models adhere to nature's basic rules, using specific computer models and mathematical methods. The outcome models can produce answers consistent with both past data and the system's inherent rules. In addition, these models can handle unfamiliar questions and gauge their own confidence in the provided answers (Zhu *et al.*, 2019).

Furthermore, it is well-known that neural networks excel at approximating continuous functions. Remarkably, it has been suggested that a neural network with just one hidden layer can accurately replicate any nonlinear continuous operator. This insight has profound implications for the capabilities of deep neural networks to process and interpret continuous operators from unstructured data. Building on this idea, researchers have extended these findings to deep neural networks, suggesting they have greater potential to address a wider array of problems.

Cuomo *et al.* (2022) conducted a comprehensive review of the existing research on PINNs. Their analysis delved into the characteristics of these networks, evaluating their strengths and weaknesses. The study highlighted various types of collocation-based PINNs, known as cPINNs, and their derivatives, including standard PINNs, conservative PINNs and the advanced variational hp-VPINN. cPINNs are designed to incorporate physical principles by using specific collocation points that enforce adherence to physical laws. The conservative PINN model ensures the preservation of conservation laws such as energy or mass. On the other hand, the variational hp-VPINN merges variational methods with high-order polynomials and adaptive meshing techniques to refine the accuracy of solutions. The research also covered different network architectures, optimization strategies, activation functions and loss functions used to customize PINNs. Although PINNs have been successfully applied in various fields, lingering theoretical challenges and opportunities for further refinement remain. The article provides an overview of PINN developments over the last four years, discussing their theoretical foundations, tackled equations, practical implementations and software tools available, with a focus on enhancing training methods and broadening their problem-solving scope.

In the past 50 years, the simulation of flow problems via numerical integration of the Navier–Stokes equations has made significant progress. However, limitations remain, such as the difficulty of incorporating chaotic data, the complexity of mesh generation and the difficulty of solving high-dimensional problems. In addition, solving inverse flow problems is frequently costly and demands the use of advanced formulas and computer codes. Literature review shows several successful attempts in predicting physical phenome using neural networks (Alkanhal, 2021; Zhang *et al.*, 2023; Shi *et al.*, 2024; Selimefendigil and Chamkha, 2020). Since neural networks are typically quick in predictions, such predictive models can be used for optimization of physical or engineering designs (Maghsoudi and Bidabadi, 2020). The neural networks can also be used to replace a numerical step in conventional simulations (Svyetlichnyy, 2018).

Considering PINNs approach, Cai *et al.* (2021a) discussed the incorporation of data and mathematical models for fluid mechanics using PINNs. The efficacy of PINNs in addressing inverse problems involving three-dimensional wake flows, supersonic flows and biomedical flows has been demonstrated. For ill-posed problems, PINNs offer a complementary approach to existing CFD solvers. Numerous research opportunities exist, including using PINNs for active flow control, predicting high Reynolds number flows through transfer learning and advancing closure models for unresolved flow dynamics. For large-scale flow simulations, efficient multi-GPU implementations are required, with hopeful parallel speed-up results suggesting potential applicability to industrial complexity problems that current CFD methods cannot address.

Cai *et al.* (2021b) explored the use of PINNs in addressing heat transfer issues, particularly under conditions where traditional computational approaches struggle. They presented examples such as convective heat transfer problems with unknown thermal boundary conditions. The study also covered the use of PINNs in power electronics and other industrial settings, highlighting their effectiveness in tackling intricate heat transfer challenges in these areas. By enhancing modeling precision and bridging the gap between experimental outcomes and simulation predictions, PINNs hold the potential to drive significant improvements in heat transfer optimization, thermal engineering and the handling of complex multiphase systems.

PINNs have been used in fluid mechanics to solve the Navier–Stokes equations (Raissi, 2018), characterize turbulent flows (Kag *et al.*, 2022; Angriman *et al.*, 2022) and simulate multiphase flows (Qiu *et al.*, 2022). Researchers have used PINNs to analyze crack propagation (Tu *et al.*, 2022), predict dynamic stress (Bolandi *et al.*, 2022), composite (Yan *et al.*, 2022) and shell (Bastek and Kochmann, 2023) structures. Furthermore, the application of PINNs in power systems (Huang and Wang, 2022), graph networks (Shukla *et al.*, 2022), molecular transport in the human brain (Zapf *et al.*, 2022) and continuum micromechanics (Henkes *et al.*, 2022) has been addressed very recently.

Lawal *et al.* (2022), using a systematic reviews and meta-analyses methodology, analyzed and assessed state-of-the-art PINNs in the computational sciences and engineering domains. The authors plan to implement a new model combining PINNs with a recurrent neural network or graph neural network via a time series data set. Using a time series data set, the authors intend to provide new models combining PINNs with a recurrent neural network or graph neural network.

Given the increasing interest and accelerated advancements in the field of PINNs, a bibliometric analysis is essential for comprehending the current state of research and determining potential future directions. Bibliometric studies are quantitative analyses of research publications that reveal patterns, trends and significant contributions to a particular research domain (Delwiche, 2018; Guo *et al.*, 2020). These studies have been used extensively to inform researchers, funding agencies and policymakers about the advancement and impact of scientific research in various disciplines (Zyoud and Fuchs-Hanusch, 2017).

In the current research, the available literature on PINNs was examined and categorized into several application categories and subcategories for the first time. The trend and impact of each category were addressed. The attention of countries to each application category was analyzed and discussed. Furthermore, the key publications and institutions that have played a pivotal role in shaping the development of PINNs are identified.

1.1 Artificial intelligence methods for physical problems

The field of scientific computation has undergone a remarkable transformation with the introduction of methods based on neural networks, especially in the area of solving PDEs. PDEs play a critical role in modeling a variety of physical phenomena, including fluid dynamics, electromagnetism and quantum mechanics. Traditional approaches to solving these equations have predominantly been numerical methods like the finite difference and finite element methods. However, these traditional methods often encounter obstacles such as elevated computational demands, challenges in dealing with complex geometries and the issue of high dimensionality in more complex problems. The advent of methods based on neural networks offers a new approach to surmount these challenges (Beck *et al.*, 2020). In some cases, a mix of CFD as a base solution and neural networks as a predictor trained on CFD data can be used (Tamaddon Jahromi *et al.*, 2022; Selimefendigil *et al.*, 2019).

1.1.1 Role of neural networks and deep learning in partial differential equations resolution. As a branch of artificial intelligence, neural networks have demonstrated remarkable efficiency in approximating intricate, nonlinear functions, making them well-suited for PDE resolution. Deep learning, a more sophisticated variant of neural networks, is adept at learning feature hierarchies, thus, being particularly effective in identifying fundamental physical laws and resolving PDEs. This potential is grounded in the universal approximation theorem, which posits that neural networks have the capability to approximate any continuous function, provided they have adequate depth and breadth.

1.1.2 Evolution of partial differential equations resolution through neural networks. The idea of using neural networks for PDE resolution is not a recent innovation. Its beginnings can be traced to the late 20th century when the potential of neural networks in approximating differential equation solutions began to be explored. The resurgence of deep learning in the current century, propelled by advancements in computational capabilities and algorithmic developments, has significantly accelerated this field. The creation of platforms like TensorFlow and PyTorch has made these potent tools more accessible, allowing a greater number of researchers to engage in experimentation and innovation.

1.1.3 Classifying neural network methods in partial differential equations resolution. The methods based on neural networks for resolving PDEs can generally be divided into three distinct categories, each with its own set of characteristics and applications (Wenshu *et al.*, 2022; Maslyaev *et al.*, 2020; Lim and Psaltis, 2022):

(1) Data-driven techniques: This technique uses neural networks to derive solutions for PDEs from partially known data, proving particularly beneficial in identifying physical equations, uncovering unknown equations and executing parameter inversion. Its strength lies in its capacity to reveal intricate patterns and connections within data, which might be unattainable through conventional methods.

(2) Physical-constraint approaches: These approaches merge physical laws into the learning processes of neural networks. By integrating governing equations and additional physical constraints into the loss function, they diminish the network's dependence on labeled data. This merging of data-driven and physics-based principles bolsters the neural network model's ability to generalize and enhances its practical value. In this strategy, the low-fidelity data generated from PINN predictions based on physical equations and high-fidelity data from field measurements can be combined for more accurate flow field reconstructions (Rui *et al.*, 2024).

(3) Physics-driven strategies: This strategy involves neural networks solving PDEs based entirely on physical laws, independent of labeled data. This approach is particularly beneficial in situations where empirical data is limited or not entirely reliable. It highlights the capacity of neural networks to function as autonomous solvers for PDEs, relying solely on the intrinsic physical principles that regulate the phenomena being analyzed. In this approach, a governing differential equation can be directly used to construct the loss function and find a solution for a physical problem (Aslam *et al.*, 2024).

Considering the neural network strategies for tackling PDEs, PINNs are classified within the "Physical-Constraint Approaches." These networks represent a fusion of data-driven methodologies and principles grounded in physics. The PINNs have the following features:

Incorporating Fundamental Physical Principles: The training process of PINNs uniquely involves embedding physical laws into the neural network's learning mechanism. This is achieved by integrating these laws within the network's loss function during training, ensuring that the network's output aligns with both the available data and the established physical laws that dictate the behavior of the phenomena in question.

1.1.4 Some physics-informed neural networks advantages. Harmonizing empirical data and physics: Diverging from methods that solely depend on empirical data, PINNs use a combination of data and physical laws. This dual-dependence equips PINNs with the ability to generate solutions that are not only more precise but also align with physical reality, a crucial factor in scenarios where data is limited or contains a significant amount of noise.

Benefits in resolving PDEs: PINNs demonstrate particular effectiveness in solving PDEs, especially given their ability to adeptly manage complex boundary conditions and the inherent nonlinear aspects of numerous PDEs. Their capability to yield solutions consistent with both empirical data and physical laws is indispensable in scientific and engineering contexts.

Broadening the scope of application: The methodology used by PINNs is widening the scope for neural network techniques in solving PDEs. This is especially relevant in areas where conventional data-driven methods are inadequate, either due to the lack of extensive data sets or the intricacies of the involved physical laws.

The aim of the present study is to analysis the scientific publications and trend of research on PINNs method. In the next section, the methodology for analysis of the related literature is discussed.

2. Methodology

2.1 Research scope

Web of Science (WoS) is a scientific database that provides access to an extensive collection of scholarly articles and research papers from various academic disciplines. This study searched the WoS database on February 27, 2023, using the below search query:

[TS = (physic* informed neural networks)] OR TS = (PINNs).

Two terms were used in the search: "physic* informed neural networks" OR "PINNs." The "TS" field, which stands for "Topic Search," was searched for these terms. The asterisk (*) following the keyword "physics" is a truncation symbol, which means that the search would include any word that begins with "physic," including "physics" and "physical."

2.2 Data analysis

Upon acquiring raw text codes from the WoS, a detailed data analysis process was set into motion, and its details are reported in Appendix 1. When it comes to keyword analysis, it is crucial to understand that a single concept can manifest in myriad ways due to the application of the stemming method (Singh and Gupta, 2017). For example, "physic inform" could alternatively be represented as "Physics informed," "Physics-informed," "Physically-informed" or "physically informed." Despite these variations, they fundamentally symbolize the same concept and should be consolidated under a single keyword. To this end, an initial step was to convert all keywords to their root form, which was then treated as the standard keyword. For instance, "physic inform" was designated as the root form, inclusive of all its variants. For additional details regarding the data analysis procedure, please refer to Appendix.

Using several important variables, the current study evaluates the productivity, impact and collaboration trends of a collection of articles. These variables shed light on various facets of the research and publication process. For example, the total publications (TP) refer to the total number of published works within a data set or subset, whereas the percentage of TPs (TP%) indicates the proportion of a particular publication type within a data set, expressed as a percentage. In addition, the study considers the total number of authors (AU) who have contributed to a data set or subset of articles, as well as the total number of institutions (Inst) linked to the authors in the data set or subset of articles.

The analysis of the number of nations (Count) represented by authors who have contributed to a data set or subset of articles is an essential aspect of the research. The study also analyzes the frequency with which articles within a data set or subset of publications have been cited by other publications (TC). The ratio of TC to TP is a useful metric for determining the average number of citations per publication within a data set or subset of publications. The study also evaluates the total engaged authors per TP (AU/TP), revealing the average number of authors involved in each publication within a specific data set or subset of publications.

The H-index (HI) is an additional crucial variable in this study, as it measures the productivity and influence of a researcher or group of academics by considering both the number of publications and the number of citations for those articles. The number of publications with the first author (AU1) is the total number of publications in a data set or subset with the same person or group as the first author. Likewise, the number of publications with the corresponding author (AUC) represents the total number of publications in a data set or subset of articles where the corresponding author is the same individual or organization.

The study concludes by examining the number of independent (Indep) and collaborative (Collab) publications, representing the total number of articles in a data set or subset of publications authored by authors working alone or in collaboration. The proportion of collaborative publications within a data set or subset of publications is represented by the ratio of collaborative publications to the total number of publications (Collab/TP).

In addition, this research precisely discerns and categorizes a diverse range of application types, as well as their associated subcategories, in the context of PINNs, as highlighted in the examined articles. The analysis then broadens its lens to scrutinize the geographical distribution of these studies and the volume of research dedicated to each type of application. Furthermore, an in-depth exploration of the temporal trends of each application type provides a holistic comprehension of the dynamism and progress within this discipline.

3. Data overview

The search was run for a certain time period, 2019 to 2022, and produced 1,209 results. Nevertheless, 213 of these data pertained to 2023, which has not yet been concluded, so they were eliminated from the total number of records, leaving 996 records pertaining to 2019 through 2022. The majority of these records are articles (84.1%), followed by proceedings papers (10.7%) and reviews (4.5%). This study adopted three types of original publications for analysis: articles, proceedings papers and articles from proceedings papers. These original sources encompassed a comprehensive data set of 946 records.

The WoS raw data and analysis results are provided as a Mendely data set as supplementary materials, which can be accessed from the following link (DOI: 10.17632/n4chxnbvjd.1) :https://data.mendeley.com/preview/n4chxnbvjd?a=33df4c79-3f37-4064-bc26-8b32e3b83085.

Table 1 presents the language distribution of 946 original publications in PINNs. The majority of the publications were written in English (99.6%), with only a small number of publications written in German, Chinese and Spanish. This finding is not surprising, given that English is the dominant language of scientific communication.

4. Results and discussions

4.1 Annual analysis

Table 2 details the characteristics of each year's published records in the field of PINNs. The number of publications has continuously climbed from 37 in 2019 to 527 in 2022, as seen in the table. Particularly, the number of publications increased significantly from 2019 to 2020, followed by a consistent increase in subsequent years. This trend indicates a growing interest in PINNs research, as well as a concomitant rise in the number of researchers and research institutes working in this topic.

The table reveals that (AU/TP) decreased from 5 in 2019 to 3 in 2020 and 2021 before increasing to 4 in 2022. The decrease in the average number of authors per publication in

Language	TP	TP (%)
English	942	99.6
German	2	0.2
Chinese	1	0.1
Spanish	1	0.1

Source: Original – from authors

Table 1. Languages represented in the analyzed publications (946 records)

2020 and 2021 could be attributed to the varying scope and focus of research projects, as well as changes in the research landscape and the wider scientific community. The increase in the average number of authors per publication in 2022 suggests that collaborations between researchers are becoming more common in PINNs.

In addition, the number of institutions contributing to the publications has increased over time, from 68 in 2019 to 656 in 2022, demonstrating that more institutions are now contributing to PINNs research. Similarly, the number of nations represented in PINNs papers has expanded from 14 in 2019 to 53 in 2022, indicating that the scope of PINNs research is growing increasingly worldwide.

The total number of citations received by publications related to PINNs varied across the years, ranging from 1,389 in 2022 to 4,466 in 2020. The peak in TC observed in 2020 could be explained by the fact that publications from 2020 addressed particularly important or novel topics, which received more attention from other researchers and therefore received more citations.

The TC/TP decreased over time, ranging from 2.6 in 2022 to 90.2 in 2019. The high average number of citations in 2019 could be attributable to the fact that it was the earliest year in the analysis, with a smaller number of articles that may have garnered greater attention at the time. On the other hand, the decrease in TC/TP in recent years may be attributable to the rise in the number of publications, which may have diminished the total influence of each article.

4.2 Countries/institutes contribution analysis

Table 3 showcases the engagement of nations in the field of PINNs. A total of 62 countries have been involved in this research. The USA and China exhibit significant activity in PINNs. The USA has contributed to 50.8% of the TP and has amassed 10,047 citations, underscoring both their prolific research volume and its widespread influence. China, on the other hand, has a contribution of 25.8% and has earned 1,925 citations, with a TC/TP value of 7.9, indicating its substantial presence in this domain. European nations, especially Germany, England, France, Switzerland, Italy and The Netherlands, are consistent contributors. Germany's engagement is especially noteworthy with its TC/TP ratio of 16.1, hinting at the high quality of its research. Collaborative research across European nations is evident, with The Netherlands, for instance, boasting a high Collab/TP% of 88.9%. In the landscape of collaborative research, the USA, despite its vast TP, has 143 collaborative outputs, suggesting its extensive global research connections. In comparison, countries such as Switzerland, Canada and Singapore place a strong emphasis on international ties, with their collaborative numbers standing out relative to their TP.

In terms of TC/TP, Singapore emerges prominently. With only 19 publications, its citation rate is a remarkable 14.3, denoting the high quality of its outputs. Similarly, Canada, Norway and Iran display commendable TC/TP values, underlining the significance of their

Table 2. Characteristics of published records per year

Year	TP	TP (%)	AU	Inst	Count	TC	TC/TP	AU/TP
2019	37	3.9	170	68	14	3,338	90.2	5
2020	113	11.9	366	146	27	4,466	39.5	3
2021	269	28.4	918	343	41	3,635	13.5	3
2022	527	55.7	1,989	656	53	1,389	2.6	4

Source: Original – from authors

Country	TP	TP (%)	TC	TC/TP	Collab	Indep	Collab/TP (%)	AU1	AUC	HI
USA	481	50.8	10,047	20.9	143	338	29.7	402	400	46
China	244	25.8	1,925	7.9	91	153	37.3	215	212	21
Germany	70	7.4	1,124	16.1	42	28	60	49	42	14
England	49	5.2	313	6.4	36	13	73.5	28	28	9
India	30	3.2	131	4.4	12	18	40	25	25	6
France	28	3	115	4.1	17	11	60.7	18	19	6
Switzerland	25	2.6	214	8.6	15	10	60	17	17	9
Canada	25	2.6	313	12.5	18	7	72	12	11	6
Italy	25	2.6	123	4.9	16	9	64	17	16	7
South Korea	24	2.5	64	2.7	10	14	41.7	19	20	5
Australia	22	2.3	129	5.9	16	6	72.7	11	9	6
Singapore	19	2	272	14.3	13	6	68.4	9	6	8
Saudi Arabia	18	1.9	161	8.9	7	11	38.9	14	15	7
Netherlands	18	1.9	105	5.8	16	2	88.9	9	7	5
Japan	14	1.5	42	3	5	9	35.7	12	11	4
Sweden	13	1.4	61	4.7	7	6	53.8	11	8	5
Denmark	13	1.4	87	6.7	5	8	38.5	11	11	4
Brazil	12	1.3	72	6	5	7	41.7	9	8	5
Norway	11	1.2	104	9.5	7	4	63.6	7	6	6
Iran	11	1.2	134	12.2	10	1	90.9	6	3	6

Source: Original – from authors

Table 3.
Top 20 countries in terms of TP related to PINNs

research in the field. Emerging players include India, South Korea, Saudi Arabia and Brazil. Although their current representation is modest, their presence suggests a budding interest and potential growth in PINNs. Japan and South Korea, both recognized for their technological advances, exhibit unique patterns in the field. Japan's contribution stands at around 1.5% of TP, while South Korea has a TC/TP of 2.7, providing insights into their varying recognition or impact on the global research stage.

In term of Research authorship, especially AU1, AUC. The USA and China, for instance, often appear in key authorship positions, indicating their central roles in the research narrative. Similarly, countries like Germany and England, despite having fewer publications, have been instrumental in leading or concluding pivotal research in the domain. The HI provides further insights. Countries such as the USA, China, some European nations, Singapore and Saudi Arabia showcase their sustained impact over time with varying HI values. For instance, while the USA's HI is 46, countries like Germany and Saudi Arabia range between 4 and 14. Some unexpected data points arise from nations like Japan and Iran. Despite Japan's renowned technological background, its HI stands at 4. Conversely, Iran, despite being lower on other metrics, boasts an HI of 6, emphasizing the consistent quality of its contributions. Notably, the collaborative nature of research is highlighted by countries like The Netherlands and Iran, with Collab/TP% figures of 88.9% and 90.9%, respectively.

The data in Table 4 provides insights into the performance of the top 20 institutes in the field of PINNs among the 924 institutes with at least one publication. Regarding TP, Brown University leads the field with 54 publications, accounting for 5.7% of the publications produced by the leading institutions. The Massachusetts Institute of Technology (MIT) ranks second with 43 publications, which accounts for 4.5% of the total. The Chinese Academy of Sciences ranks third with 25 publications, accounting for 2.6% of all publications. The Pacific Northwest National Laboratory and Penn State University have 22 and 18 publications, representing 2.3% and 1.9% of the total number of publications in PINNs.

Table 4.
Statistical analysis of the top 30 institutions in the field of PINNs by TP

Institution	TP	TP (%)	TC	TC/TP	Collab	Indep	Collab/TP (%)	AU1	AUC	HI
Brown Univ	54	5.7	5,198	96.3	20	34	37	42	37	27
MIT	43	4.5	1,286	29.9	21	22	48.8	18	18	19
Chinese Acad Sci	25	2.6	153	6.1	7	18	28	15	15	7
Pacific Northwest Natl Lab	22	2.3	1,369	62.2	6	16	27.3	7	7	14
Penn State Univ	18	1.9	125	6.9	3	15	16.7	15	15	4
Purdue Univ	18	1.9	201	11.2	5	13	27.8	12	11	7
Univ Penn	18	1.9	2,887	160.4	4	14	22.2	10	12	12
Univ Illinois	17	1.8	204	12	4	13	23.5	8	9	8
Stanford Univ	17	1.8	204	12	6	11	35.3	11	13	8
Univ Chinese Acad Sci	16	1.7	137	8.6	6	10	37.5	14	0	6
Univ Arizona	15	1.6	126	8.4	5	10	33.3	9	10	6
Swiss Fed Inst Technol	14	1.5	121	8.6	7	7	50	10	8	7
Univ Notre Dame	14	1.5	744	53.1	4	10	28.6	9	10	10
Tsinghua Univ	14	1.5	30	2.1	8	6	57.1	10	12	4
Hong Kong Polytech Univ	13	1.4	52	4	5	8	38.5	5	5	4
Univ Cent Florida	13	1.4	170	13.1	1	12	7.7	12	12	9
Northeastern Univ	12	1.3	279	23.3	2	10	16.7	12	12	6
Cornell Univ	11	1.2	62	5.6	4	7	36.4	10	6	4
Univ Michigan	11	1.2	317	28.8	2	9	18.2	7	7	5
Shanghai Jiao Tong Univ	10	1.1	50	5	4	6	40	5	7	3

Source: Original – from authors

With 5,198 TC, Brown University once again ranks first, followed by Penn State University with 2,887 citations. Third place is the Pacific Northwest National Laboratory with 1,369 citations, followed by the MIT and University univ Notre Dame with 1,286 and 744 citations, respectively. This indicates that Brown University's research output in PINNs has a significant quantitative and qualitative impact on the field.

The TC/TP measures the impact per publication on average. In this regard, Brown University has an impressive average of 96.3 citations per publication, demonstrating their work's high relevance and influence in PINNs. Despite having fewer publications (18), the University of Pennsylvania has the highest TC/TP ratio of 160.4, indicating that their work has a significant impact per publication. MIT's TC/TP ratio is 29.9, while the Pacific Northwest National Laboratory's is 62.2, and the Chinese Academy of Sciences' is 6.1.

With 21 collaborative publications, MIT is in first place, followed closely by Brown University with 20 collaborative publications. The Chinese Academy of Sciences and the Swiss Federal Institutes of Technology each have seven collaborative publications, while Penn State University has only four. Brown University continues to lead the pack in terms of Indep with 34 publications. MIT's research output is well-balanced, with 22 independent publications accounting for slightly more than half of its total output. The Chinese Academy of Sciences has 18 Indep, whereas the Pacific Northwest National Laboratory and Penn State University each have 16 and 15, respectively.

The five universities with the highest ratio of Collab/TP are Tsinghua University (57.1%), Swiss Federal Institute of Technology (50%), MIT (48.8%), Shanghai Jiao Tong University (40%) and Hong Kong Polytechnic University (38.5%). These institutions place a significant emphasis on interdisciplinary research and collaborative endeavors, which contributes to their status as preeminent research universities.

Regarding the number of publications with the first author, Brown University stands out with 42 publications, nearly double the number of publications with the first author at MIT, which has 18 publications with the first author. The Chinese Academy of Sciences and Penn State University each have 15 publications with the first author, whereas the Pacific Northwest National Laboratory has the fewest, with only seven.

Brown University leads in the number of publications, with the corresponding author with 37, followed by MIT with 18 publications. Each of the Chinese Academy of Sciences and Penn State University has published 15 works with the corresponding author. This indicates that researchers at Brown University are not only conducting research but also communicating and supervising the work.

Finally, HI measures the significance and efficacy of a university's research output. Brown University's HI of 27, the highest in PINNs, reflects its productive and influential research efforts. The Pacific Northwest National Laboratory has an HI of 14, while MIT has an HI of 19. The Chinese Academy of Sciences and Penn State University have H-indices of 7 and 4, respectively, indicating that their research output is less influential than that of other leading institutions.

4.3 Journals analysis

Table 5 provides a comprehensive overview of the 20 leading journals in the field of PINNs among the 472 journals in PINNs. The *Journal of Computational Physics* ranks first on the list with the maximum total number of publications (57) and a substantial 6% proportion of the total number of publications in the field of PINNs. This demonstrates the journal's prominent standing and dedication to disseminating cutting-edge PINNs research. In addition, the journal has received a total of 3,541 citations, highlighting its significance and influence within the academic community.

After the *Journal of Computational Physics*, *Computer Methods in Applied Mechanics and Engineering* rates second with 48 TP, accounting for 5.1% of the field's TP. This journal also boasts a high TC count of 2,009, indicating that its published articles have received

Journal name	TP	TP (%)	TC	HI
Journal of Computational Physics	57	6	3,541	18
Computer Methods in Applied Mechanics and Engineering	48	5.1	2,009	21
Physics of Fluids	23	2.4	218	9
Scientific Reports	14	1.5	70	4
IFAC Papersonline	12	1.3	6	1
IEEE Access	10	1.1	54	4
Energies	9	1	37	2
Sensors	9	1	21	3
Engineering Applications of Artificial Intelligence	8	0.8	120	5
Nonlinear Dynamics	8	0.8	153	5
Journal of Petroleum Science and Engineering	8	0.8	68	4
Computers and Fluids	8	0.8	64	3
Computers and Structures	8	0.8	46	5
Physical Review Fluids	8	0.8	197	6
Chaos Solitons and Fractals	7	0.7	79	5
Neurocomputing	7	0.7	107	4
IEEE Transactions on Geoscience and Remote Sensing	7	0.7	30	2
Siam Journal on Scientific Computing	7	0.7	443	6
Chaos	7	0.7	20	3
Applied Sciences-Basel	7	0.7	54	3

Source: Original – from authors

Table 5. Leading 20 journals in terms of TP within the PINNs domain

considerable attention and recognition. The journal's HI of 21 underscores the significance of its research contributions to PINNs.

Physics of Fluids, which ranks third, has a total of 23 publications, or 2.4% of all PINNs publications. Despite the fact that its TC count of 218 is significantly lower than the top two journals, it still retains a respectable position within the PINNs research community.

Scientific Reports and IFAC Papersonline have 14 and 12 TP, accounting for 1.5% and 1.3% of the TP in the discipline, respectively. However, these journals have relatively lower TC counts: *Scientific Reports* with 70 citations and *IFAC Papers* online with only six citations, reflecting the relatively lower influence of their publications within the PINNs domain. *IEEE Access*, *Energies* and *Sensors* have each published 10 and 9 articles, respectively. These journals have made contributions to the field of PINNs with citation counts spanning from 21 to 54, indicative of their modest influence on the research community.

It is important to recognize that this list does not encompass all journals active in the field of artificial intelligence and simulations. The omission is due to the focus of some journals on different aspects of the field. For instance, there are studies on the support vector machine (Shadloo, 2020), optimization of engineering designs using artificial neural networks (Zeeshan *et al.*, 2023, Pakatchian *et al.*, 2020), the use of deep neural networks for predicting material properties (Kim and Moon, 2022) and inverse problems (Löhner *et al.*, 2021). These represent alternative approaches to modeling and simulation in physical problems. Consequently, several journals that contribute to artificial intelligence and physical simulations might not be included in this list.

Figure 1 displays the publication tendencies of the top five journals in the field of PINNs over time (2019–2022). *Journal of Computational Physics*, *Computer Methods in Applied Mechanics and Engineering*, *Physics of Fluids, Scientific Reports* and *IFAC PapersOnline* are among the journals included.

There is a distinct upward trend in the number of publications in each of the five journals from 2019 to 2022, indicating a growing interest in PINNs research. *The Journal of Computational Physics* saw the greatest increase in publications, from three in 2019 to 29 in 2022. This rapid expansion may be attributed to the journal's emphasis on computational methods and its relevance to developing and applying PINNs in various research fields. Similarly, the number of articles published in *Computer Methods in Applied Mechanics and Engineering* increased significantly from 10 in 2020 to 24 in 2022. This journal's multidisciplinary nature, with an emphasis on computational mechanics, makes it an ideal venue for research on PINNs, which frequently entails interdisciplinary collaboration among researchers from various disciplines.

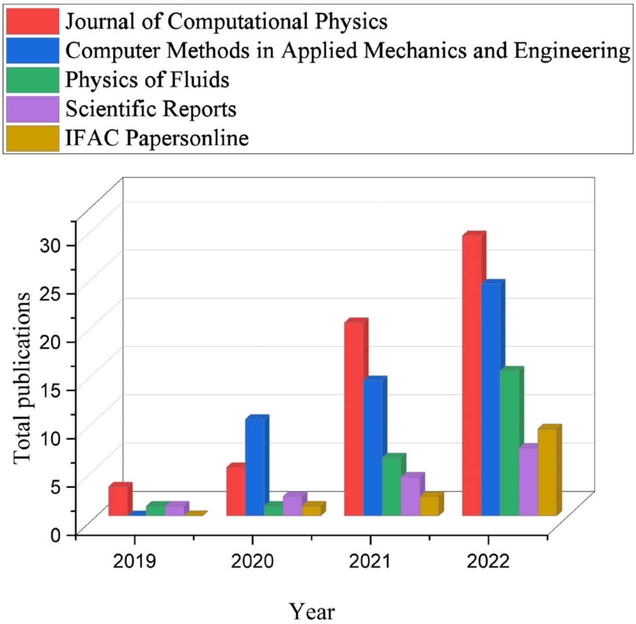
Physics of Fluids publications related to PINNs increased from 1 in 2019 to 15 in 2022. The journal's focus on fluid mechanics is pertinent to studying fluid flow and heat transfer mechanisms using PINNs, which is crucial to developing and applying these neural networks.

The number of PINNs articles published in the multidisciplinary journal *Scientific Reports* increased from 1 in 2019 to 7 in 2022. This expansion may be attributable to the journal's broad scope, which attracts research from diverse disciplines, such as implementing PINNs for various problems.

The number of PINNs publications published by *IFAC PapersOnline*, which publishes papers on automatic control and related disciplines, increased from zero in 2019 to nine in 2022. Researchers are investigating the application of control techniques and systems engineering in the context of PINNs, as evidenced by this expansion.

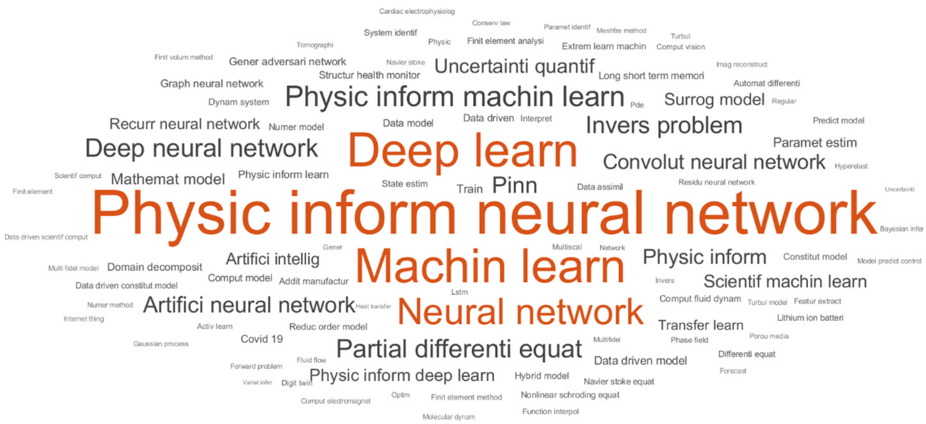
4.4 Keywords analysis and physics-informed neural networks trending

A keyword cloud map, depicted in Figure 2, represents the commonly used keywords in PINNs. The font size denotes the keywords' importance, whereas the larger the font, the



Note: The top five journals were selected based on the TP associated with PINNS
Source: (Original - From authors)

Figure 1.
Temporal publication trends for the top five journals in PINNs



Note: The size of each keyword describes its frequency in published WoS records associated with PINNs
Source: (Original - From authors)

Figure 2.
Root keywords cloud map illustrating the PINNs research landscape for the top 100 root keywords

greater the importance of the keyword. In addition, repeated keywords also indicate their importance. The center of the map, highlighted in red color, represents the physics-informed neural network keyword, which is a significant component of the research on PINNs. The map contains 100 commonly used keywords related to PINNs.

Table 6 displays the top 20 frequently used keywords among the 100 commonly used keywords shown in Figure 2. Physic Inform Neural Network has the highest TP of 283, representing 29.9% of the TP. "Machin Learn" has the second-highest TP value of 152 and a TP% of 16.1%, making it the second most published keyword. It also has the highest TC value of 3,846 and an HI of 20, indicating that it is highly influential and widely cited in physics-informed neural networks.

The keyword with the lowest TP is Paramet Estim, with only 13 publications, accounting for 1.4% of the TP. Machin Learn, Deep Learn and Neural Network are the second, third and fourth most important keywords, respectively, with TP% values of 16.1%, 16.0% and 9.5%, respectively.

In terms of TC, Machin Learn has the highest TC of 3,846, followed by Physic Inform Neural Network with a TC of 3,215. Deep Learn has the third-highest TC with 1,535 citations. The keyword with the lowest TC is Mathemat Model, with only 32 citations. HI is a measure of the productivity and impact of a keyword. Physic Inform Neural Network has the highest HI of 29, followed by Deep Learn and Machin Learn, with HI values of 21 and 20, respectively.

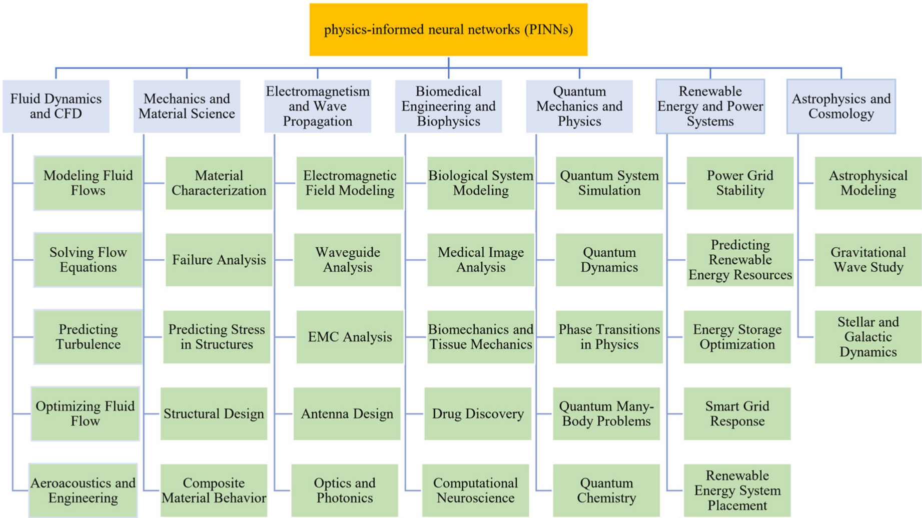
4.5 In-depth application categories analysis and trending

In the following section, we delve into a comprehensive exploration of the various applications of PINNs. These applications have been divided into seven primary categories, each featuring related subcategories. Figure 3 provides a visual illustration, encapsulating the distribution and relationship of these functional categories and their respective subcategories within PINNs.

Table 6. Top 20 root keywords associated with PINNs

Keyword root	TP	TP (%)	TC	HI
Physic inform neural network	283	29.9	3,215	29
Machin learn	152	16.1	3,846	20
Deep learn	151	16	1,535	21
Neural network	90	9.5	930	14
Physic inform machin learn	56	5.9	796	14
Partial differenti equat	50	5.3	607	11
Deep neural network	45	4.8	1,360	14
Invers problem	38	4	770	12
Artifici neural network	34	3.6	352	9
Convolut neural network	31	3.3	179	7
Physic inform	30	3.2	837	9
Uncertainti quantif	28	3	684	11
Scientif machin learn	22	2.3	596	12
Artifici intellig	20	2.1	115	7
Surrog model	20	2.1	248	8
Physic inform deep learn	20	2.1	210	6
Mathemat model	17	1.8	32	4
Recurr neural network	17	1.8	116	5
Transfer learn	14	1.5	312	6
Paramet estim	13	1.4	168	4

Source: Original – from authors



Note: Publications frequency for each category is reported in Table 7
Source: (Original - From authors)

Figure 3.
Suggested categories and subcategories based on analysis of 946 original articles on PINNs

Table 7 is organized according to these categories, providing a concise overview of the distribution of published research within the field of PINNs. Table 7 is derived from a thorough examination of 946 original articles focused on PINNs. These articles have been categorized into seven main domains, further divided into various subcategories. It should be noted that an article may fall into several categories. Thus, the sum of the numbers in Table 7 could equal or exceed the total number of articles.

The category “Fluid Dynamics and Computational Fluid Dynamics (CFD)” represents a substantial field in physics and engineering that is concerned with the study of fluids (gases, plasmas and liquids) in motion. In our data set of 946 PINNs articles, this category, divided into five distinct subcategories, showcases a broad spectrum of research focus within the realm of fluid dynamics. The subcategory “Modeling Fluid Flows” accounts for the highest proportion (39%) within this category. This suggests that many researchers are exploring how PINNs can help simulate fluid flows more efficiently and accurately. These models are essential in various industries, including aviation, automotive and environmental science, where understanding fluid behavior can drive innovation and enhance performance. “Solving Flow Equations” is the next dominant subcategory, featuring 26% of articles. This indicates a keen interest in using PINNs to tackle the mathematical intricacies of fluid flow equations, like Navier-Stokes equations, which describe the motion of fluid substances. Such research could lead to breakthroughs in the precise prediction and control of fluid dynamics systems. The subcategory “Predicting Turbulence” with 16% of articles, underscores the challenges and importance of turbulence prediction in fluid dynamics. Turbulence, despite its complexity, is a common phenomenon in fluid flows, affecting aerodynamic performance, energy efficiency and heat transfer processes. Researchers seem to be leveraging PINNs to enhance turbulence modeling and prediction. Some studies consider using PINNs in such as outdoor airflow (Rui *et al.*, 2023) and indoor airflow field using PINNs (Wei and Ooka, 2023) and neural networks (Gao *et al.*, 2024).

Categories	Subcategory	TP	TP (%)
Fluid dynamics and CFD	Modeling fluid flows	316	39
	Solving flow equations	213	26
	Predicting turbulence	127	16
	Optimizing fluid flow	109	14
	Aeroacoustics and engineering	39	5
Mechanics and material science	Material characterization	66	40
	Failure analysis	33	20
	Predicting stress in structures	32	20
	Structural design	25	15
	Composite material behavior	8	5
Electromagnetism and wave propagation	Electromagnetic field modeling	26	23
	Waveguide analysis	25	23
	EMC analysis	21	19
	Antenna design	20	18
	Optics and photonics	19	17
Biomedical engineering and biophysics	Biological system modeling	50	41
	Medical image analysis	26	21
	Biomechanics and tissue mechanics	18	15
	Drug discovery	14	12
	Computational neuroscience	13	11
Quantum mechanics and physics	Quantum system simulation	38	51
	Quantum dynamics	13	17
	Phase transitions in Physics	11	15
	Quantum many-body problems	10	13
	Quantum chemistry	3	4
Renewable energy and power systems	Power grid stability	6	30
	Predicting renewable energy resources	5	25
	Energy storage optimization	4	20
	Smart grid response	3	15
	Renewable energy system placement	2	10
Astrophysics and cosmology	Astrophysical modeling	4	57
	Gravitational wave study	2	29
	Stellar and galactic dynamics	1	14

Source: Original – from authors

Table 7.
Application categories and subcategories in the PINNs

Recent studies like (Pioch *et al.*, 2023, Xiao *et al.*, 2023, Hanrahan *et al.*, 2023) highlight the innovative approaches being taken within these subcategories. For instance, (Pioch *et al.*, 2023) describes the use of various Reynolds-averaged Navier–Stokes (RANS) models in a PINN framework to predict turbulent flows more accurately, potentially enhancing our understanding and control of such complex systems (Pioch *et al.*, 2023). Similarly, (Xiao *et al.*, 2023) explores an improved PINN framework specifically for the challenging Rayleigh-Taylor turbulence, suggesting a broader applicability of PINNs in handling complex, multi-scale flow problems (Xiao *et al.*, 2023). Finally, the research in Section 3 demonstrates the capability of PINNs to infer critical flow features from limited data, which could revolutionize experimental approaches in fluid dynamics (Hanrahan *et al.*, 2023). These examples underscore the dynamic nature of research in this field and the potential for PINNs to make significant contributions to fluid dynamics and beyond.

"Optimizing Fluid Flow," featuring 14% of articles, reflects endeavors to use PINNs to optimize fluid flow in various systems, from pipelines to airfoils. This could significantly enhance system efficiency and reduce energy consumption. Finally, "Aeroacoustics and

Engineering," despite having the least proportion of articles (5%), is a niche area focusing on the interaction between fluid flow and sound. Given its crucial applications in reducing noise pollution from vehicles, wind turbines and aeronautical equipment, this might signify an opportunity for future research.

The category "Mechanics and Material Science" is central to the field of physics, engineering and industry, dealing with the behavior of materials under various conditions. It focuses on the behavior of materials under different physical conditions, and how we can design and analyze structures using these materials. In the data set of 946 original articles on PINNs, five subcategories are delineated within this broader category. The "Material Characterization" subcategory stands out, boasting the highest proportion of published articles (40%) in this category. This demonstrates a robust interest in leveraging PINNs to improve our understanding of material properties. This involves understanding the complex physical properties of various materials, including elasticity, hardness, ductility and tensile strength. The insights gained from these studies directly influence the design and manufacturing processes across industries ranging from aerospace and automotive to electronics and biomedical engineering.

Next, "Failure Analysis" has 20% of articles. This subcategory underscores the significant role of PINNs in predicting potential failures in materials and mechanical systems. By better understanding how materials fail under different conditions, such as extreme temperatures, loads or corrosive environments, researchers and engineers can proactively design materials and structures to avoid such failures. The subcategory "Predicting Stress in Structures" contributes 20% of articles. Stress analysis is crucial in the design process of any structure, from bridges and buildings to aircraft and automobile components. PINNs are used to predict the effects of different stress conditions on structures, leading to safer and more efficient designs. "Structural Design" follows next with 15% of articles. It underlines the use of PINNs in streamlining the design process, from optimizing design parameters to simulating the performance of the structure under various conditions. This can lead to significant time and cost savings in the design process. Finally, "Composite Material Behavior" is the least represented subcategory, with only 5% of articles. Composite materials combine two or more materials to create superior properties, and they are widely used in industries like aerospace, automotive and sports equipment. The lower count suggests that there are significant opportunities for future research, specifically in leveraging PINNs to understand and predict the behavior of these complex materials. The structures and beams have also been simulated using PINNS as discussed in (Kapoor *et al.*, 2024, Kapoor *et al.*, 2023).

The "Electromagnetism and Wave Propagation" category in this study represents an integral part of modern physics, encompassing research areas that form the basis for much of today's technology, such as telecommunications, radar systems and optical devices. PINNs are increasingly being applied to these domains, offering new ways to model, analyze and optimize electromagnetic systems. The subcategory "Electromagnetic Field Modeling" with 26% of articles leads in this category. This area includes research related to modeling and simulating electromagnetic fields, essential for various applications like MRI scanners, wireless communication or electronic component design. This interest demonstrates the importance of accurate electromagnetic field modeling in technological innovation. "Waveguide Analysis" has 25% of articles.

Waveguides are structures that guide waves, like electromagnetic or sound waves, from one point to another. In telecommunications and radar systems, waveguide designs directly impact the performance and efficiency of the systems. Therefore, PINNs' application in waveguide analysis shows a commitment to advancing our understanding and improving these critical systems. The subcategory "EMC Analysis" contributes 21% of articles. Electromagnetic compatibility (EMC) is the ability of electrical devices to function correctly in their electromagnetic environment without introducing intolerable electromagnetic disturbances. Understanding and ensuring EMC is becoming increasingly crucial as our world becomes more interconnected and reliant on electronics. "Antenna Design" holds 20% of articles. Antennas are essential for broadcasting and receiving signals in devices ranging from cell phones to satellites. Using PINNs in antenna design can lead to better performance, more efficient designs and the creation of antennas for new frequencies or applications. The least represented subcategory is "Optics and Photonics," with 19% of articles. These fields study the behavior and properties of light and its interaction with matter. Their applications are vast, from fiber-optic communications and laser systems to medical devices and even quantum computing. The lower representation suggests the potential for more exploration and application of PINNs in this area.

The "Biomedical Engineering and Biophysics" category encompasses significant research areas in biology and medicine where PINNs are being implemented to uncover new findings and improve processes. "Biological System Modeling," with 30% of articles, stands out as the most explored subcategory in this area. These studies revolve around constructing mathematical and computational representations of biological systems, like organs, cellular networks or even entire organisms. Using PINNs in such modeling can lead to more accurate simulations, a better understanding of system dynamics and predictive tools for studying diseases or testing therapies. "Medical Image Analysis" has 16% of articles. This subcategory demonstrates the use of PINNs in analyzing medical imaging data, such as X-rays, MRI scans and CT scans, for better diagnostics and understanding of various medical conditions. This fusion of AI with medical imaging can potentially revolutionize healthcare, offering quicker, more precise diagnoses and personalized treatment plans. With 11% of articles, "Biomechanics and Tissue Mechanics" investigates the mechanical properties of biological tissues and their response to various forces. Accurate modeling of these mechanics is crucial for surgical planning, prosthetics design and understanding disease progression like cancer metastasis.

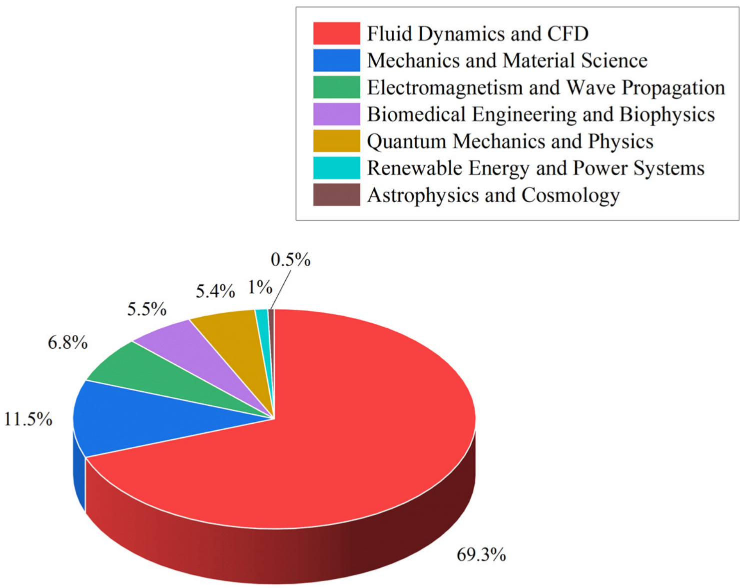
PINNs can provide a novel approach to capturing these complex behaviors. "Drug Discovery," with 8% of articles, denotes the use of PINNs in streamlining the drug discovery process. From predicting drug interactions to modeling biological pathways affected by potential drugs, PINNs could accelerate drug development, making it less expensive and more effective. Finally, "Computational Neuroscience," with 8% of articles, uses PINNs to model and understand the functioning of the brain and nervous system. The lesser representation in this subcategory may suggest an area ripe for further exploration.

The latter part of the table provides an overview of how PINNs are applied to different fields, such as quantum mechanics, renewable energy and astrophysics. "Quantum System Simulation" leads the "Quantum Mechanics and Physics" category with 60% of articles. It illustrates how PINNs can offer fresh insights into quantum systems by simulating their complex dynamics. This is particularly helpful in systems that exhibit quantum entanglement or superposition, which are otherwise computationally challenging to model. "Quantum Dynamics," "Phase Transitions in Physics," "Quantum Many-Body Problems" and "Quantum Chemistry" have fewer articles, ranging from 21% to 5%, indicating that these areas are relatively less explored. Despite their lower representation, they encompass significant quantum phenomena that can benefit immensely from PINNs, such as understanding temporal changes in quantum states, predicting phase transitions, solving many-body problems and modeling chemical reactions at the quantum level.

The “Renewable Energy and Power Systems” category explores how PINNs can optimize and manage energy systems. “Power Grid Stability,” with 30% of articles, indicates the potential of PINNs in maintaining the stability and reliability of power grids. The other subcategories, including “Predicting Renewable Energy Resources,” “Energy Storage Optimization,” “Smart Grid Response” and “Renewable Energy System Placement,” with 10% to 25% of articles each, demonstrate the application of PINNs in optimizing energy use, predicting renewable energy output and deciding optimal locations for renewable energy systems. Finally, the “Astrophysics and Cosmology” category contains articles on “Astrophysical Modeling,” “Gravitational Wave Study” and “Stellar and Galactic Dynamics,” ranging from 40% to 10% of articles. These illustrate the use of PINNs in studying large-scale cosmic phenomena, predicting gravitational waves – a breakthrough in modern astrophysics, and understanding the dynamics of stars and galaxies. A few articles could indicate that these areas are still at the beginning stages of implementing PINNs in their research.

Figure 4 presents the distribution of the PINNs articles in each application category by percentage, offering a broader perspective of the current research landscape and the interest of researchers in various areas of science and engineering.

The category of “Fluid Dynamics and CFD” is notably prevalent in the research landscape, comprising a substantial 69.3% of all articles. This indicates a marked emphasis on research applying PINNs to issues in fluid dynamics. Areas of focus include the modeling of fluid movements, addressing flow equations, forecasting turbulence, optimizing the flow of fluids and studying aeroacoustics. The high percentage signifies the versatility and effectiveness of PINNs in handling the complexities of fluid dynamics problems such as nonlinearity, turbulence and multiphase flows. The “Mechanics and Material Science”



Note: Most of the publications are in the “Fluid Dynamics and CFD” category, followed by Mechanics and Material Science
Source: (Original - From authors)

Figure 4.
Distribution of published articles in application categories

category, accounting for 11.5% of the articles, is the second most researched area. Research within this category primarily concentrates on exploring the properties of materials, forecasting stress in different structures, examining mechanisms of failure and investigating the behavior of composite materials. The prominent representation of this category underscores the significant role that PINNs can play in enhancing the safety and effectiveness of structures and materials across a range of sectors, including construction and aerospace.

The "Electromagnetism and Wave Propagation" category follows with 6.8% of the articles, reflecting the application of PINNs in understanding and manipulating electromagnetic fields. The studies in this category encompass electromagnetic field modeling, waveguide analysis, EMC analysis, antenna design and optics and photonics. This reflects the significant implications of PINNs in various sectors, including telecommunications, energy and health care.

The "Biomedical Engineering and Biophysics" and "Quantum Mechanics and Physics" categories represent 5.5% and 5.4% of the articles, respectively. These indicate that PINNs are being increasingly recognized in both traditional and modern physics realms, spanning from biological system modeling and drug discovery to quantum system simulations and quantum chemistry. Finally, the "Renewable Energy and Power Systems" and "Astrophysics and Cosmology" categories have the least representation, with 1% and 0.5% of the total articles, respectively. While this may suggest that these areas are still emerging in the context of PINN applications, it also highlights significant opportunities for future research, considering the global urgency for renewable energy solutions and the ongoing cosmic explorations in astrophysics.

Figure 5 shows a comprehensive view of the trending research areas in PINNs over the years, from 2019 to 2022. It allows us to observe how interest and research in different categories have evolved over time.

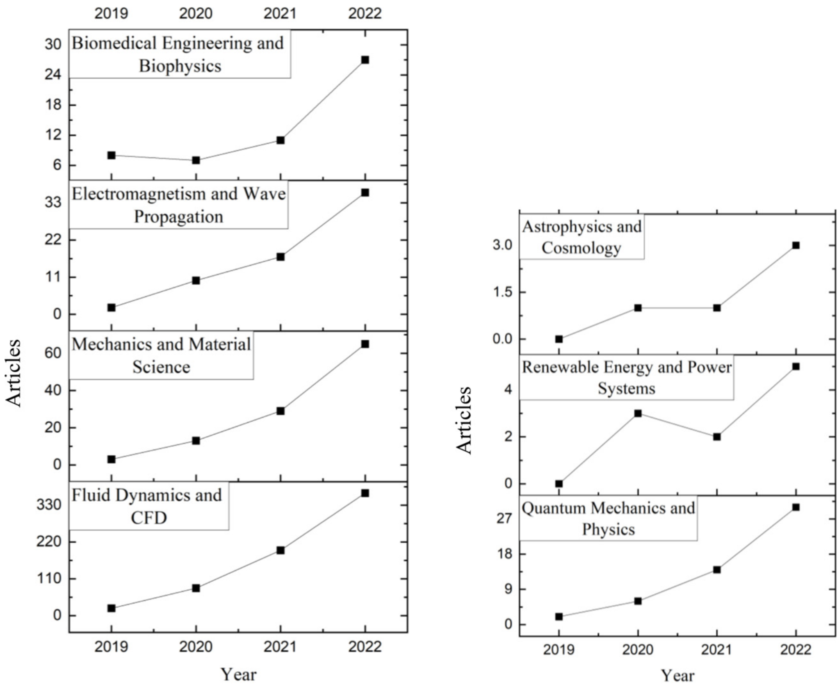
The "Fluid Dynamics and CFD" category consistently remains the most active area of research over the years. Starting with just 22 articles in 2019, it shows a substantial leap to 82 in 2020, followed by a more than doubling in 2021 to 195 articles. The trend continues with an impressive rise to 366 articles in 2022. The exponential growth in this category underscores the increasing adoption and recognition of the effectiveness of PINNs in addressing complex problems in fluid dynamics and CFD.

Next, "Mechanics and Material Science" is another category that has seen significant growth. Beginning with just three articles in 2019, there is a consistent increase in the number of publications in this area, reaching 65 by 2022. This trend indicates an expanding interest in applying PINNs to understand the characteristics of different materials and the mechanics of structures, which has wide-ranging applications in various industries.

The "Electromagnetism and Wave Propagation" category also follows a similar upward trajectory, even though at a slower pace. With a jump from two articles in 2019 to 36 articles in 2022, the increasing interest in this field signifies the versatility of PINNs in modeling electromagnetic fields and wave propagation, which has significant implications in sectors such as telecommunications, electronics and health care.

The growth in the "Biomedical Engineering and Biophysics" category is steady but slower compared to other categories, suggesting a more cautious adoption of PINNs in these areas, possibly due to the complexities and sensitivities involved in biomedical applications. However, the consistent growth from 8 to 27 articles between 2019 and 2022 shows increasing interest and potential in applying PINNs in biological system modeling, drug discovery and other related fields.

The "Quantum Mechanics and Physics" category has also shown consistent growth, albeit at a slower rate. Starting with just two articles in 2019, the category has seen a steady



Notes: The primary categories are chosen from Table 7. Each subplot illustrates the annual publication count for one of the primary categories. All application categories for PINNs exhibit a generally upward trend in publications
Source: (Original - From authors)

Figure 5.
Trends in the Evolution of seven application categories for PINNs over time

rise to 30 articles by 2022. The growth, although modest, highlights the expanding role of PINNs in addressing complex quantum problems, ranging from quantum system simulation to quantum chemistry.

The "Renewable Energy and Power Systems" category shows a small but steady increase from 0 to 5 articles between 2019 and 2022. This trend signifies the emerging interest in the application of PINNs in renewable energy and power systems. Given the urgent global call for clean and sustainable energy sources, this area will likely witness more robust growth in the future.

Finally, the "Astrophysics and Cosmology" category, though starting from zero in 2019, sees a slow but steady increase to three articles by 2022. While this category remains the least researched, it highlights an emerging and exciting area for PINNs' application in studying astrophysical phenomena and cosmic dynamics.

Figure 6 illustrates PINNs application categories in the six most productive countries: the USA, China, Germany, England, India and France. This analysis offers insights into the international trends and focus areas in PINNs research.

The category Fluid Dynamics and CFD has seen the highest number of publications. The USA has published 319 articles, which is a testament to the country's strong research base and interest in this field. China, too, has shown substantial interest in Fluid Dynamics and CFD, contributing 180 articles showcasing the high importance placed on these areas.

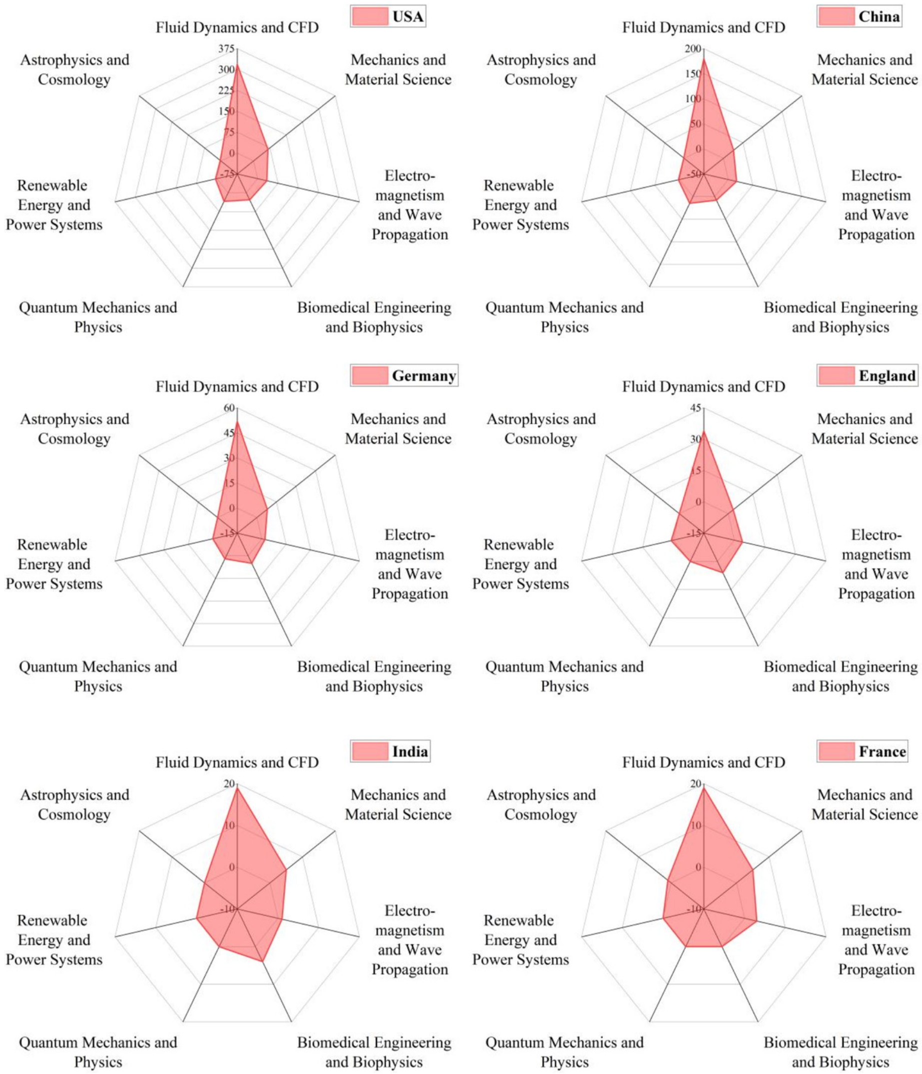


Figure 6.
Radar charts depicting seven application categories across the most productive countries: the USA, China, Germany, England, India and France

Note: The selection of the top six countries was determined by their cumulative publications in the domain of PINNs
Source: (Original - From authors)

Germany, England, India and France follow, indicating a worldwide research focus on the understanding and simulation of fluid dynamics which are essential in numerous industries, including aerospace, automotive, energy and more.

Mechanics and Material Science is the second most active area of research, with the USA again leading with 66 articles. China trails behind with 26 articles, demonstrating its strong foothold in this crucial field. It highlights the importance of understanding materials' physical properties and behavior under different conditions in industries like construction, manufacturing and more.

Electromagnetism and Wave Propagation also have significant contributions from the USA and China. This research area is critical for advancements in communication technologies, radar systems and even health-care technologies. Biomedical Engineering and Biophysics is another active area of research, particularly in the USA and China. In this category, using PINNs can lead to significant breakthroughs in understanding complex biological systems, medical imaging, drug discovery and even neural activity, demonstrating the intersection of healthcare and artificial intelligence.

In the category of Quantum Mechanics and Physics, the USA again leads, illustrating its strong commitment to quantum research. Quantum mechanics can revolutionize our understanding of the universe and could lead to advancements in computing, cryptography and material science.

Renewable Energy and Power Systems have fewer publications, with the USA contributing five articles. Despite the low numbers, this shows the increasing interest in using PINNs to optimize energy systems, improve renewable energy forecasts and manage power grid stability, aligning with global sustainability efforts. Astrophysics and Cosmology, a complex field dealing with the universe's origins and evolution, also shows limited activity, with the USA leading with three articles.

4.6 Most cited publications

In this section, the publications with over 100 citations are selected. PINNs are a very recent topic, so these publications have received significant attention. Table 8 shows a list of articles that have gained over 100 citations in the PINNs field. Figure 7 provides the topic and main features of the top ten most cited publications in PINNs. It is important to recognize that earlier-published articles often accumulate more citations. Therefore, the relative trend in citation numbers could serve as another metric to identify emerging publications. Given that the field of PINNs is relatively recent, those publications that have garnered significant citations are likely to be the most influential in the field.

The top 20 papers can be divided into PINN Frameworks and Applications (Raissi *et al.*, 2019; Lu *et al.*, 2021b, Meng and Karniadakis, 2020; Jagtap and Karniadakis, 2021; Meng *et al.*, 2020), PINNs in Fluid Mechanics (Samaniego *et al.*, 2020; Raissi *et al.*, 2020; Sun *et al.*, 2020; Mao *et al.*, 2020; Jin *et al.*, 2021), PINNs in Specialized Applications (Pang *et al.*, 2019; Goswami *et al.*, 2020; Jagtap *et al.*, 2020b, Kissas *et al.*, 2020; Oviedo *et al.*, 2019; Chen *et al.*, 2020) and Enhancements and extensions of PINNs (Jagtap *et al.*, 2020a, Zhang *et al.*, 2019; Yang *et al.*, 2021; Yang and Perdikaris, 2019; Pun *et al.*, 2019).

PINN Frameworks and Applications includes papers that focus on the development and application of PINNs and associated frameworks (Raissi *et al.*, 2019; Lu *et al.*, 2021b, Meng and Karniadakis, 2020; Jagtap and Karniadakis, 2021; Meng *et al.*, 2020). These studies are centered around solving differential equations, estimating functions and tackling time-dependent PDEs. The foundational paper (Raissi *et al.*, 2019) introduces PINNs as a deep learning framework to solve forward and inverse problems involving nonlinear PDEs. DeepXDE (Lu *et al.*, 2021b) presents a deep-learning library designed to solve differential equations using PINNs. The neural network (Meng and Karniadakis, 2020) uses multi-fidelity data to learn the inverse PDE problems through function approximation. PINNs improve the overall performance and robustness of the model. Extended PINNs (XPINNs) (Jagtap and Karniadakis, 2021) offer a generalized space-time decomposition framework using a deep learning framework for solving nonlinear PDEs, allowing for more efficient

Ref.	Title	PY	TC
Raissi *et al.* (2019)	"Physics-informed neural networks: a deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations"	2019	2,098
Samaniego *et al.* (2020)	"An energy approach to the solution of partial differential equations in computational mechanics via machine learning: concepts, implementation and applications"	2020	490
Raissi *et al.* (2020)	"Hidden fluid mechanics: learning velocity and pressure fields from flow visualizations"	2020	394
Lu *et al.* (2021b)	"Deepxde: a deep learning library for solving differential equations"	2021	258
Sun *et al.* (2020)	"Surrogate modeling for fluid flows based on physics-constrained deep learning without simulation data"	2020	190
Mao *et al.* (2020)	"Physics-informed neural networks for high-speed flows"	2020	189
Pang *et al.* (2019)	"fPINNs: fractional physics-informed neural networks"	2019	170
Goswami *et al.* (2020)	"Transfer learning enhanced physics informed neural network for phase-field modeling of fracture"	2020	165
Jagtap *et al.* (2020a)	"Adaptive activation functions accelerate convergence in deep and physics-informed neural networks"	2020	159
Jagtap *et al.* (2020b)	"Conservative physics-informed neural networks on discrete domains for conservation laws: applications to forward and inverse problems"	2020	141
Jin *et al.* (2021)	"NSFnets (Navier–Stokes flow nets): physics-informed neural networks for the incompressible Navier–Stokes equations"	2021	131
Zhang *et al.* (2019)	"Quantifying total uncertainty in physics-informed neural networks for solving forward and inverse stochastic problems"	2019	127
Yang *et al.* (2021)	"B-PINNs: Bayesian physics-informed neural networks for forward and inverse PDE problems with noisy data"	2021	126
Kissas *et al.* (2020)	"Machine learning in cardiovascular flows modeling: predicting arterial blood pressure from non-invasive 4d flow MRI data using physics-informed neural networks"	2020	125
Yang and Perdikaris (2019)	"Adversarial uncertainty quantification in physics-informed neural networks"	2019	124
Meng and Karniadakis (2020)	"A composite neural network that learns from multi-fidelity data: application to function approximation and inverse PDE problems"	2020	123
Pun *et al.* (2019)	"Physically informed artificial neural networks for atomistic modeling of materials"	2019	117
Oviedo *et al.* (2019)	"Fast and interpretable classification of small x-ray diffraction datasets using data augmentation and deep neural networks"	2019	115
Chen *et al.* (2020)	"Physics-informed neural networks for inverse problems in nano-optics and metamaterials"	2020	107
Jagtap and Karniadakis (2021)	"Extended physics-informed neural networks (xpinns): a generalized space-time domain decomposition based deep learning framework for nonlinear partial differential equations"	2020	105
Meng *et al.* (2020)	"PPINN: parareal physics-informed neural network for time-dependent PDEs"	2020	102

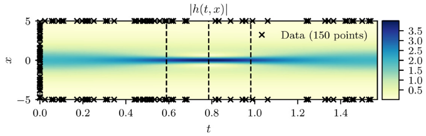
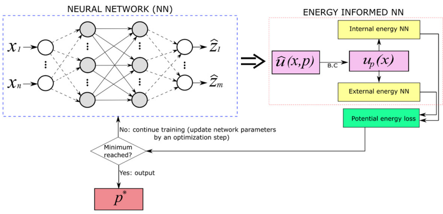
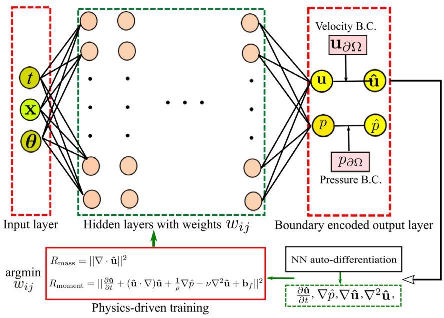
Source: Original – from authors

Table 8. Articles that have garnered over 100 citations in the PINNs field

and accurate solutions. Finally, the Parareal PINN (Meng *et al.*, 2020) emphasizes solutions for time-dependent PDEs. This approach integrates parallel-in-time integration techniques, aiming to enhance the scalability and efficiency of solutions derived fromPINNs.

PINNs in Fluid Mechanics cover papers that apply PINNs and machine learning techniques to fluid mechanics (Samaniego *et al.*, 2020; Raissi *et al.*, 2020; Sun *et al.*, 2020; Mao *et al.*, 2020; Jin *et al.*, 2021). These works demonstrate the potential of PINNs in solving complex fluid dynamics problems. The energy approach in (Samaniego *et al.*, 2020) solves

PDEs in computational mechanics using machine learning, while (Raissi *et al.*, 2020) learns velocity and pressure fields from flow visualizations. Surrogate modeling (Sun *et al.*, 2020) uses physics-constrained deep learning without simulation data for fluid flows and (Mao *et al.*, 2020) applies PINNs to high-speed flows. Navier–Stokes Flow Nets (NSFNet)

Ref.	Representative image	Description
(Raissi et al., 2019) Permission form Elsevier		A showcase of the effectiveness of PINNs in predicting solutions to the Schrödinger equation. It juxtaposes the predicted and exact solutions, demonstrating PINNs' capability in modeling complex nonlinear behaviors with limited initial data.
(Samaniego et al., 2020) Permission form Elsevier		A visual representation of the Deep Energy Method's comprehensive process, particularly focusing on the intricate task of boundary conditions implementation in problem-solving through Deep Neural Networks.
(Raissi et al., 2020)		Illustration of a 2D simulation of fluid flowing past a cylinder, detailing the concentration field and velocity profiles. It highlights different boundary conditions, the scalar injection area, and two distinct training domains, one flower-shaped and another rectangular, with specific locations where velocity data is provided.
(Lu et al., 2021b)		A schematic representation of a PINN solving a diffusion equation with mixed boundary conditions. It also illustrates the set of residual points for the equation and boundary/initial conditions.
(Sun et al., 2020) Permission form Elsevier		A structured, fully-connected neural network diagram within the data-free, physics-constrained deep learning framework for surrogate modeling of fluid flows. The network, with inputs of time, spatial coordinates, and variable parameters, minimizes the residuals of Navier-Stokes equations for training. It simultaneously upholds fluid flow conservation laws and satisfies specified initial and boundary conditions.

(*continued*)

Figure 7.
Top 10 most cited articles and their topics

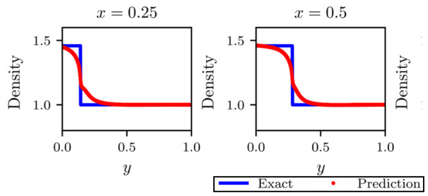
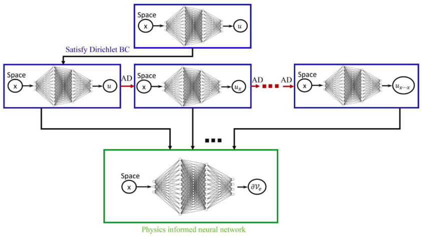
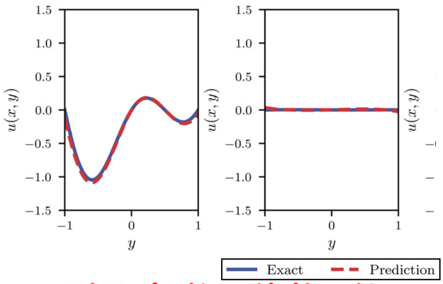
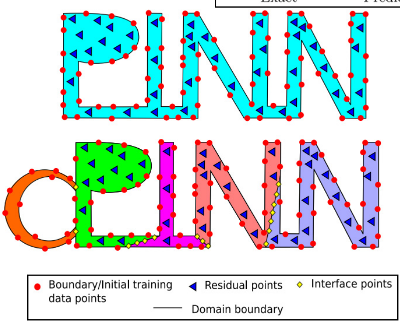
(Mao et al., 2020) Permission form Elsevier		A forward problem for the oblique shock wave issue. It compares the density with the exact solution at various x locations for two random and clustered points.
(Pang et al., 2019)	Two training point distributions for 1D time-dependent diffusion equations within PINNs. The left diagram displays a lattice-like arrangement coinciding with a finite difference grid, while the right side depicts scattered training points derived from a quasi-random sequence.	
(Goswami et al., 2020) Permission form Elsevier		The proposed framework for an energy-based PINNs for phase-field modeling of fracture. The diagram includes the computational graph utilized for training the PINN, demonstrating the sequence of steps from neural network construction to the minimization of variational energy, highlighting how boundary conditions are handled and energy variation is computed.
(Jagtap et al., 2020a) Permission form Elsevier		A solution to the Helmholtz equation was obtained with an adaptive activation function. A comparison between the exact solution and the PINN solution using fixed and adaptive 'tanh' activation functions after 3600 iterations at a specific location.
(Jagtap et al., 2020b) Permission form Elsevier		A schematic representation of both PINN and cPINN methods. It shows how cPINN, unlike PINN, splits the domain into multiple sub-domains, each employing a different neural network. This setup enables different network architectures, efficient parallelization, and a choice of hyperparameters tailored to the solution regularity in each sub-domain.

Figure 7.

(Jin *et al.*, 2021) uses PINNs for the incompressible Navier–Stokes equations, providing an efficient and accurate solution for fluid flow problems.

PINNs in Specialized Applications focus on specialized applications of PINNs (Pang *et al.*, 2019; Goswami *et al.*, 2020; Jagtap *et al.*, 2020b, Kissas *et al.*, 2020; Oviedo *et al.*, 2019; Chen *et al.*, 2020). Fractional PINNs (Pang *et al.*, 2019) are proposed for problems involving fractional-order PDEs, while (Goswami *et al.*, 2020) uses transfer learning to enhance PINNs for phase-field modeling of fracture. Conservative PINNs (Jagtap *et al.*, 2020b) are applied to discrete domains for conservation laws, and Kissas *et al.* (2020) predict arterial blood pressure using PINNs on noninvasive 4D flow MRI data. Fast and interpretable classification of small x-ray diffraction data sets (Oviedo *et al.*, 2019) is achieved using data augmentation and deep neural networks. Finally, (Chen *et al.*, 2020) applies PINNs to inverse problems in nano-optics and metamaterials, demonstrating the versatility of PINNs in various fields.

Enhancements and extensions of PINNs deal with enhancements and uncertainty quantification in PINNs (Jagtap *et al.*, 2020a, Zhang *et al.*, 2019; Yang *et al.*, 2021; Yang and Perdikaris, 2019; Pun *et al.*, 2019). Adaptive activation functions (Jagtap *et al.*, 2020a) accelerate convergence in deep and physics-informed neural networks, improving the efficiency of learning complex relationships. Uncertainty quantification is crucial in understanding the reliability of model predictions, and papers by Zhang *et al.* (2019) and Yang and Perdikaris (2019) focus on quantifying uncertainty in PINNs when solving forward and inverse stochastic problems. Bayesian methods are used in Yang *et al.* (2021), introducing Bayesian PINNs for solving inverse and forward PDE problems involving noisy data. Physically informed artificial neural networks (Pun *et al.*, 2019) are used for atomistic modeling of materials, demonstrating the potential for incorporating physical constraints into various types of neural networks.

5. Conclusions

In this comprehensive examination of the literature on PINNs, a total of 996 records from the WoS database spanning the years 2019 to 2022 were retrieved. Initially, an overview of the field was presented, highlighting the contributions of institutes and countries. Subsequently, the literature records were carefully categorized into seven main categories and 33 subcategories. The distribution of application categories and subcategories was extensively analyzed and reported. In addition, there was an in-depth discussion on the evolution of the main categories over time. A significant portion of the analysis was devoted to examining the primary areas of focus of the countries most actively engaged in each application category. The principal insights from this study can be summarized in the following points:

- The publications were categorized into seven primary domains, each with various subcategories: Fluid Dynamics and CFD, Mechanics and Material Science, Electromagnetism and Wave Propagation, Biomedical Engineering and Biophysics, Quantum Mechanics and Physics, Renewable Energy and Power Systems and Astrophysics and Cosmology.
- "Fluid Dynamics and CFD" dominate PINNs research, accounting for 69.3% of total articles. Among five distinct subcategories, "Modeling Fluid Flows" represents the largest area of focus (39% of articles), followed by "Solving Flow Equations" (26%). The smallest subcategory, "Aeroacoustics and Engineering" (5%), provides a unique area for potential future exploration. The interest in this category has seen exponential growth, with articles rising from 22 in 2019 to 366 in 2022, indicating an increasing recognition of the effectiveness of PINNs in addressing complex problems in fluid dynamics and CFD.

- The “Mechanics and Material Science” category, encompassing 11.5% of PINNs articles, has been marked by considerable growth, from three articles in 2019 to 65 in 2022. The research is segmented into five subcategories, with “Material Characterization” leading (40% of articles), indicating a strong interest in using PINNs to improve understanding of material properties. “Failure Analysis” and “Predicting Stress in Structures” comprise 20% of articles, reflecting PINNs’ significant role in predicting potential failures and stress effects. Despite “Composite Material Behavior” having only 5% of articles, it signals a ripe area for future research in understanding complex materials.
- Within the category of “Electromagnetism and Wave Propagation,” which encompasses critical research areas such as telecommunications, radar systems and optical devices, there is a growing trend of using PINNs. Notably, this category includes significant subcategories like “Electromagnetic Field Modeling,” accounting for 26% of the articles, and “Waveguide Analysis,” making up 25%. These subcategories underscore their pivotal role in technological advancements. Although “Optics and Photonics” is less represented, constituting 19% of the research, it holds promising potential for future applications of PINNs. The number of articles in this category has grown from two in 2019 to 36 in 2022, emphasizing the rising interest and the versatility of PINNs in modeling electromagnetic fields and wave propagation, which is critical for advancements in various sectors, including healthcare and communications.
- The “Biomedical Engineering and Biophysics” category, where PINNs are increasingly applied, covers areas like “Biological System Modeling” (30% of articles) and “Medical Image Analysis” (16%), indicating the promising potential for PINNs in healthcare. Other research realms such as “Quantum Mechanics and Physics,” “Renewable Energy and Power Systems” and “Astrophysics and Cosmology” collectively suggest a growing recognition of PINNs across diverse fields, despite representing only 5.5%, 5.4%, 1% and 0.5% of articles, respectively. The trend analysis shows steady growth in each category from 2019 to 2022, indicating the expanding role of PINNs in addressing complex problems in these sectors and suggesting significant potential for future research.
- The PINNs field has leaders such as the United States, China, Germany, England and India, underlining the importance of international cooperation. Top research institutions include Brown University, MIT and the Chinese Academy of Sciences, with Brown University leading in terms of TP and Indep publications, as well as those with a first or corresponding author. The primary channels for sharing developments are journals like the *Journal of Computational Physics and Computer Methods in Applied Mechanics and Engineering*. Engineering, Computer Science and Physics were identified as top subject areas, showing PINNs’ interdisciplinary potential. Frequently used terms in PINNs research include “Physics Informed Neural Network,” “Deep Learning” and “Machine Learning.” The most cited articles are related to Frameworks and Applications for PINNs and PINNs in Fluid Mechanics. Recent advancements have shown PINNs’ promise for multiphase flows despite some computational efficiency issues. Future research can potentially enhance PINNs’ effectiveness and precision.
- The six most productive countries in PINNs research are the USA, China, Germany, England, India and France, with Fluid Dynamics and CFD as the most published category. The USA leads with 319 articles, followed by China with 180. The Mechanics

and Material Science category is the second most active, led by the USA with 66 articles. Biomedical Engineering and Biophysics; Electromagnetism and Wave Propagation; and Quantum Mechanics and Physics also see significant contributions, especially from the USA and China. Despite fewer publications, the categories Renewable Energy and Power Systems and Astrophysics and Cosmology indicate emerging interest, signifying potential growth areas for PINNs applications.

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Further reading

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Appendix

The raw data was extracted from WoS, where a snapshot of the raw data, particularly a recorded segment related to PINNs, drawn from WoS is depicted in Figure A1. To transmute this raw data into insightful and statistically significant reports, a Python script was designed and used.

This Python script holds a central role in sifting through the raw data to extract valuable information and process it accordingly. However, it is important to underline that certain phases of the analysis, manual inspection and intervention were applied. The process commences with the application of filters to the data based on distinct criteria, such as types of publications and publication years. The script then classifies the data into a dictionary comprising diverse elements like keywords, institutes, countries and journal names, among others. Moreover, evaluation indices are computed for each element, serving as the foundation for ranking the items and generating connectivity matrices.

However, it is worth highlighting that text format inconsistencies and irregularities in author affiliations often call for manual corrections. Similarly, the standardization of abbreviations and resolving similar keywords frequently demand human intervention. Furthermore, certain types of analysis, such as identifying the types of PINNs, resist automation and call for hands-on human analysis.

The procedure for analysis using python can be explained by steps illustrated in Figure A2. These steps are:

Title: Data-driven and physics-informed deep learning operators for solution of heat conduction equation with parametric heat source
Author(s): Koric, S (Koric, Seid); Abueidda, DW (Abueidda, Diab W.)
Source: INTERNATIONAL JOURNAL OF HEAT AND MASS TRANSFER **Volume:** 203 **Article Number:** 123809 **DOI:** 10.1016/j.ijheatmasstransfer.2022.123809 **Early Access Date:** DEC 2022 **Published:** APR 2023
Times Cited in Web of Science Core Collection: 1
Total Times Cited: 1
Usage Count (Last 180 days): 19
Usage Count (Since 2013): 19
Cited Reference Count: 23
Abstract: Deep neural networks as universal approximators of partial differential equations (PDEs) have attracted attention in numerous scientific and technical circles with the introduction of Physics-informed Neural Networks (PINNs). However, in most existing approaches, PINN can only provide solutions for defined input parameters, such as source terms, loads, boundaries, and initial conditions. Any modification in such parameters necessitates retraining or transfer learning. Classical numerical techniques are no excep-tion, as each new input parameter value necessitates a new independent simulation. Unlike PINNs, which approximate solution functions, DeepONet approximates linear and nonlinear PDE solution operators by using parametric functions (infinite-dimensional objects) as inputs and mapping them to different PDE solution function output spaces. We devise, apply, and compare data-driven and physics-informed Deep-ONet models to solve the heat conduction (Poisson's) equation, one of the most common PDEs in science and engineering, using the variable and spatially multi-dimensional source term as its parameter. We provide novel computational insights into the DeepONet learning process of PDE solution with spatially multi-dimensional parametric input functions. We also show that, after being adequately trained, the pro-posed frameworks can reliably and almost instantly predict the parametric solution while being orders of magnitude faster than classical numerical solvers and without any additional training.(c) 2022 Elsevier Ltd. All rights reserved.
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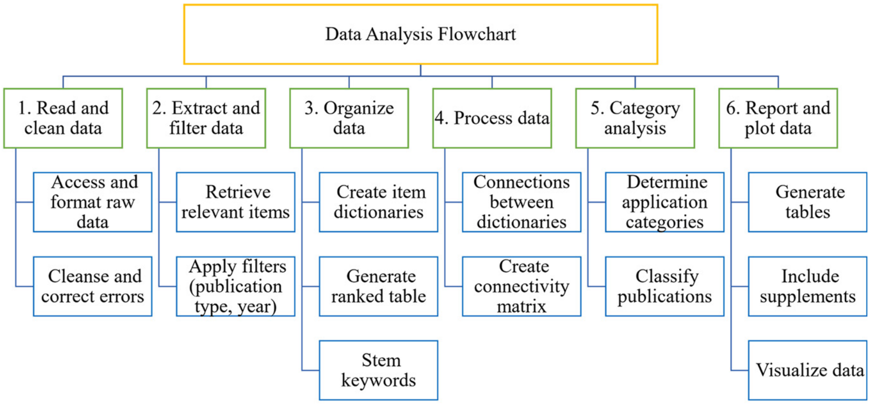
Note: The presented image illustrates a segment of data from a specific record, providing a glance into the content encompassed within WoS
Source: Original – from authors

Figure A1.
An example of unprocessed data obtained from WoS



Source: Original – from authors

Figure A2.
An illustration depicting the stages involved in processing data

(1) Import necessary libraries and modules for data analysis, such as pandas, numpy and matplotlib. Import raw text and make a database to store the data in an organized structure.
(2) Read and clean data:
   - Access the raw bibliographic data using Python's file-handling capabilities.
   - Use appropriate methods from pandas or other libraries to import the data and create a dataframe.
   - Apply data cleaning techniques to correct any errors or inconsistencies in the data, such as removing duplicates, fixing formatting issues and handling missing values.

(3) Extract and filter data:
- Identify the relevant information required for the analysis, such as publication titles, authors, publication years and keywords.
- Use pandas dataframe manipulation techniques to extract and filter the data based on specific criteria, such as publication types or a specific range of publication years.

(4) Organize data:
- Create dictionaries or data structures to store and organize the extracted data.
- Generate a rank table based on predefined criteria, such as the number of citations, publication frequency or any other relevant metric.
- Implement techniques like stemming or lemmatization to transform keywords to their root form, enhancing consistency in the analysis.

(5) Process data:
- Establish connections or cross-references between the different data dictionaries or structures, enabling further analysis and exploration.
- Create a connectivity matrix to identify relationships and co-occurrences between the data set's authors, keywords or other relevant entities.

(6) Category analysis::
- Determine the application categories or subject areas.
- Apply appropriate classification techniques, such as topic modeling, to categorize the publications based on their content.

(7) Generate reports and visualizations:
- Use python's data manipulation and visualization libraries to generate informative tables summarizing the analyzed data, including relevant statistics and metrics.
- Include supplementary materials, such as plots, charts or graphs, to visually represent the analyzed bibliographic data.
- Export the generated reports and visualizations in a suitable format, such as PDF or HTML, for further analysis or sharing with others.

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