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# Convection heat transfer in enclosures with inner bodies: A review on single and two-phase nanofluid models

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ABSTRACT

This review peruses a comprehensive implementation of inner bodies involved in cavities filled with regular and nanofluids for both natural and mixed convection modes. The topic of nanofluid acquired its importance in the heat transfer field about two decades ago. Many experimental studies have been conducted to establish accurate correlations between the nanoparticle's specifications (volume fraction, size, shape, and type) and the thermophysical properties (such as dynamic viscosity and thermal conductivity). However, most of the theoretical investigations focused on the use of nanoparticles in various geometrical conduits and enclosures. The review covers the implementation of nanofluid in cavities involving inner bodies using single and two-phase models. It is found that about 34% of the total reviewed studies have considered the nanofluid inside the cavities involving inner cylinders. The authors of this review have concluded that the extra numerical cost of the two-phase models has curbed the researcher's use of the single-phase model. It is found that the studies that adopt the two-phase model are only 19% of the total nanofluid studies. The two-phase model gave an essential detail about the collection of the nanoparticles on the surfaces of the inner bodies (primarily stationary bodies) and the segments of the undulations of wavy walls of the cavities. This phenomenon lowers the exchange of heat transfer.

## 1. Introduction

### 1.1. Cavities with inner bodies

Inner bodies are involved inside cavities for two purposes: to enhance and excite the heat transfer and simulate some engineering processes, such as heat exchangers. The presence of inner bodies can be found in natural and mixed convection modes. Nanofluids can also be used in such cavities to enhance heat exchange. The topic of nanofluid acquired its importance in the heat transfer field about two decades ago. Experimental studies have been conducted to establish real correlations between the nanoparticle's specifications (volume fraction, size, shape, and type) and the thermophysical properties (especially the dynamic viscosity and the thermal conductivity), while most of the theoretical studies have discovered the wide use of nanofluids in various applications in natural and mixed convection modes, especially in industrial engineering [1–3]. To predict the properties of nanofluids, it is necessary to use an accurate correlation, which is the main challenge in theoretical studies. Early studies showed that thermal conductivity is the key parameter that enhances the heat transfer process. Thus, the single-phase model was widely used and endorsed [4]. The two-phase model, which Buongiorno published in 2006, added a comprehensive

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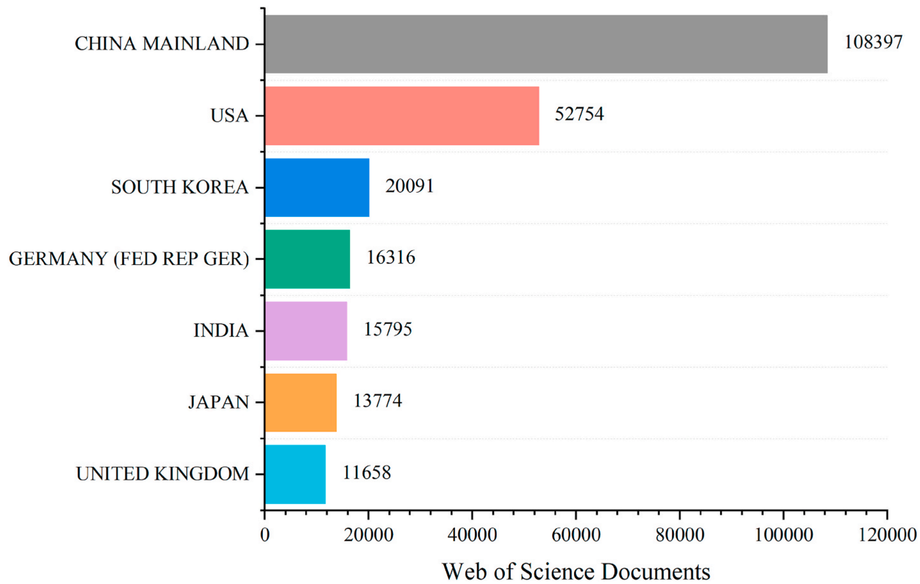



**Fig. 1.** The Web of Science documents (among 266,983 documents) in time period: 2017–2021 in research area of nanoscience & nanotechnology from InCites dataset updated 3 April 2022.

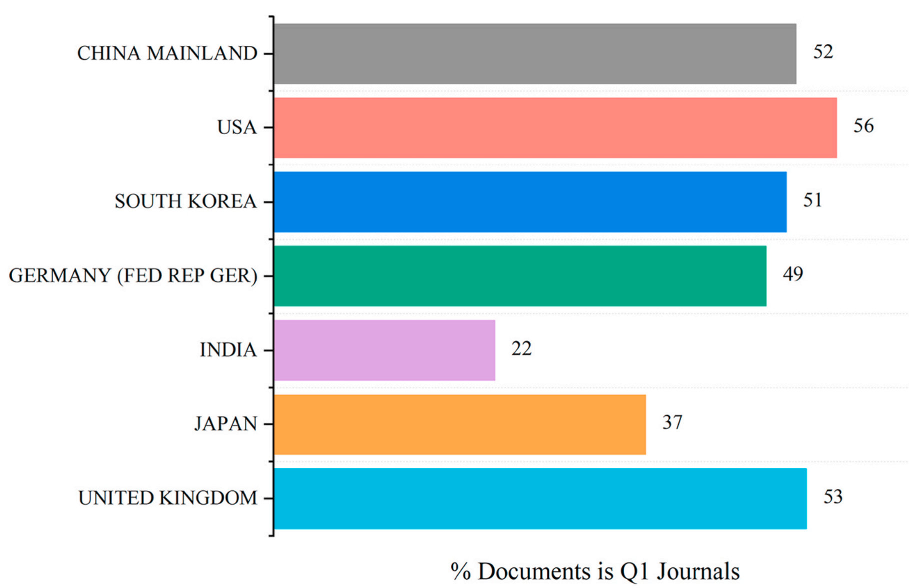



**Fig. 2.** The percentage of publications in Q1 journals based on Web of Science documents in the time period: 2017–2021 in the research area of nanoscience & nanotechnology from InCites dataset updated 3 April 2022.

sight to the motion of nanoparticles regarding the base fluid. Two of seven mechanisms were distinguished to be more effective in diffusing the nanoparticles: diffusion due to temperature gradient (thermophoresis) and diffusion due to the random motion of nanoparticles (Brownian motion). Together with the thermal conductivity, these two mechanisms have interpreted many unusual deteriorations of heat transfer using nanofluids [5]. This paper reviews the works regarding the cavities involving inner bodies filled with regular fluid or nanofluids and solved using single and two-phase models in both free and mixed convective heat transfer. Accordingly, in each category, the papers are reviewed chronologically. Several recent studies have reviewed nanofluids' type, thermophysical properties, and heat transfer applications [6]. Some of physical aspects of heat transfer in enclosures such as the flow regime and numerical method has also been discussed in Saha et al. [7]. However, the single-two phase approaches along with the shape of enclosures has not been reviewed for convection heat transfer of nanofluid. The present study addresses the convection heat transfer in enclosures considering the utilized modeling approaches (single- and two-phase models) in detail (see Tables 3 and 4).

### 1.2. *Bibliography of nanoscience & nanotechnology*

Fig. 1 depicts the number of published documents during the past five years in the field of nanoscience & nanotechnology in the top seven countries worldwide. China published the most documents with 108,397, followed by the USA with 52,754 documents. Fig. 2 illustrates the Q1 publications in the field. This figure reveals that the USA published most of the Q1 papers (56%), followed closely by China with 52%. Although China published twice of documents as compared to the USA, the number of Q1 papers is almost similar. Considering the topic of nanofluids, there are 19,456 documents, which is 7.3% of the total documents in the field of nanoscience & nanotechnology. The top ten

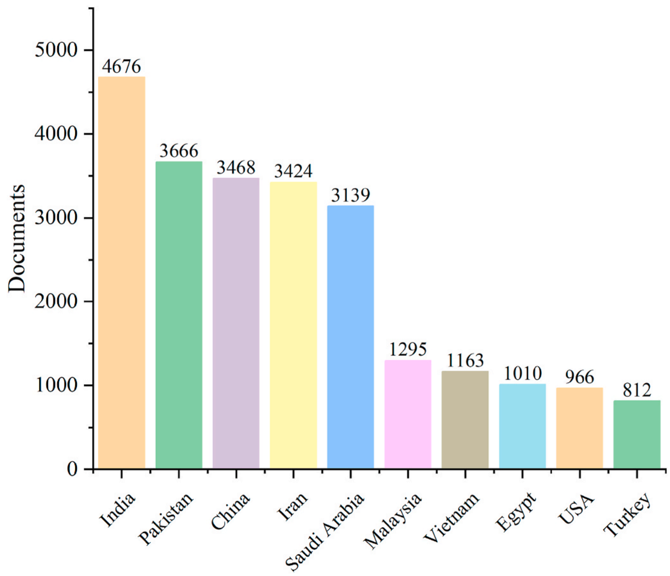



**Fig. 3.** The top ten world countries with most documents on the topic of nanofluids: Web of Science documents (19,456 documents) in time period: 2017–2021 dataset updated 3 April 2022.

countries with the most documents on the topic of nanofluids are shown in Fig. 3. India, Pakistan, and China are the top three countries on the list. Among these 19,456 documents, about 180 documents are concerned with this review's topic, which will be analyzed in the following sections.

## 2. Regular fluids

In this section, the natural and mixed convection studies that used regular or regular fluid are reviewed.

### *2.1. Natural convection with internal bodies-regular fluids (NCIB-RF)*

#### *2.1.1. Preliminary illustration*

Natural convection or "free convection" is a flow that represents the motion of different fluids, including water, air, molten metal, and others. The fluid flow in natural convection problems is induced by natural means such as buoyancy; buoyancy forces may arise in a fluid with a gradient of density in which body forces are proportional to the density. Since the natural convection induced velocities are relatively low, the heat transfer coefficient encountered in these cases is usually low. Natural convection is an interesting scientific topic for which many aspects need further research. It is attracted a vast deal of attention from researchers and scientists due to its wide applications either in nature, such as natural phenomena like atmospheric and oceanic currents, bio-heat transfer, greenhouse effects, and heat transfer in stellar atmospheres; or in engineering, such as in the design of equipment for electronic cooling [8], insulation of aircraft cabins [9], nuclear reactors [10], and heating and ventilation control of buildings [11] and so on. In this review section, we will introduce various studies where the natural convection generated inside enclosures with internal bodies directly or indirectly affecting heat transfer through the enclosure's fluid. Because of its unique and important applications, such as microwave heating, electrochemical batteries, solar to thermal energy conversion, geophysics, and nuclear reactors, natural convection heat transfer induced by internal heating or absorption has received and continues to receive much attention.

#### *2.1.2. NCIB-RF: (1992–2000)*

In 1992 a numerical study on natural convective heat transfer from a uniformly heated horizontal cylinder placed in a large air-filled rectangular enclosure. The dynamics of the flow and the thermal behavior are studied at various heat fluxes applied to the cylinder [12]. The average Nusselt number correlations on the Rayleigh number were obtained using numerical data. Natural convection in the annulus can be considered as a convective heat transfer in a cylinder with an inner cylindrical body. Table 1 lists some crucial studies on natural convection heat transfer in enclosures filled with clear fluid and containing an obstacle. Some of these studies have been discussed in more detail in this section. Thus, Burns and Stewart [9] studied natural convection in cylindrical annulus filled with a porous material with inner heat generation elements under the influence of isothermal cylinders and permeable outer cylinders. Authors revealed that a permeable outer border augmented the heat transfer at the surface of the impermeable inner body. Stewart and Burns [10] investigated the same problem in another work, but the outer border was impermeable, and the inner boundary was permeable. Such modification also characterizes heat transfer enhancement (see Table 2).

Moreover, a rise in the thermal generation reflects essential fluid penetration in a porous region with a reduction of the boundary layer thickness along the outer border. Sawada et al. [13] examined MHD natural convective heat transfer of magnetic liquid experimentally in a concentric annulus under the influence of a hot inner cylinder, cold outer cylinder, and an applied magnetic field. The authors showed that the magnetic field could manage the free convection of magnetic liquid.

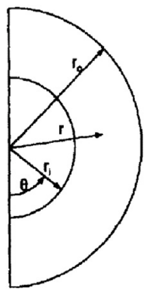
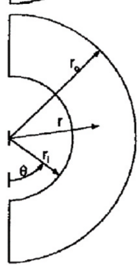
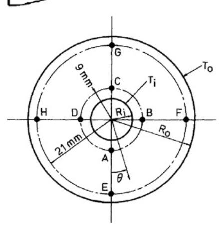
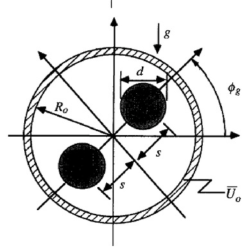
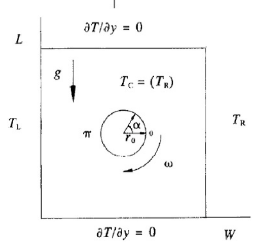
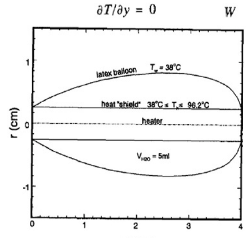
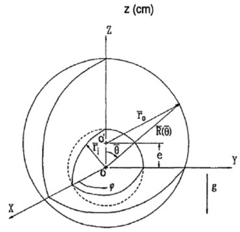
Wang et al. [11] performed some numerical computations in square enclosures suited with a vertical plate to evaluate the amount of the average Nusselt number. The obtained results show that the enhancement of the Nusselt number can be achieved by narrowing the distance between the building walls (enclosure walls) and the fitted inner plate.

Ho et al. [14] numerically investigated the natural convection of air in circular cavities with internal differentially heated cylinders under an impact of external convection conditions. It was ascertained that the addition of convective boundary conditions at the outer border with inner isothermal cylinders characterizes essential intensification of convective flow within the chamber. The natural convection of Newtonian fluid in a differentially heated square chamber under the effect of an inner cool spinning cylinder was numerically studied by Fu et al. [15]. Using primitive variables and finite element techniques, the authors found that taking into account the different temperatures at vertical walls of the chamber, the direction of inner cylinder rotation has an essential influence on natural convection augmentation within the chamber. Lei and Kleinstreuer [16] simulated the natural convection of water within a balloon in the presence of an inner cylindrical heated element. Using the finite volume method, the authors revealed that the thermal boundary conditions at an inner element and the chamber orientation can be considered beneficial parameters for designing the local hyperthermia device treatment of undesirable tissue.

Natural convection heat transfer between a heated horizontal cylinder positioned concentrically within a square enclosure, and numerical solutions are provided in Moukalled and Acharya [17]. In either a rectangular or an air-filled circular enclosure, calculations of the coupled convection and conduction heat transfer rate for a solid cylinder are performed by Liu et al. [18], where In the aforementioned domain, the temperature distribution and fluid flow analysis are obtained. The results showed that the usual procedure of setting a uniform heat flux boundary condition at the interface might induce some errors and inaccuracies in the solution. A numerical analysis was carried out for free convective heat transfer for the working fluid of air and from two vertically separated horizontal cylinders which were confined to a rectangular cavity. The heat sink temperature was also set for the vertical finite conductance walls and horizontal. The result shows that along with the interface of vertical wall-fluid, the Nusselt number is a complex function of thermal conductivity ratio and the Raleigh number [19].

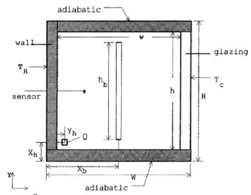
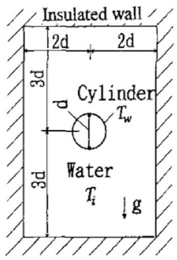
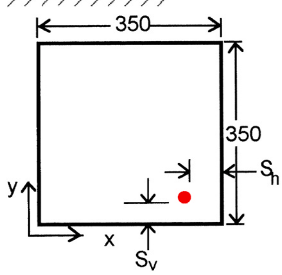
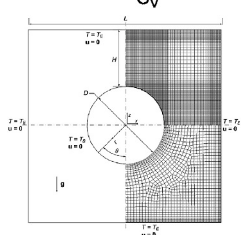
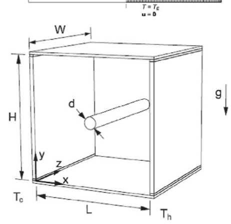
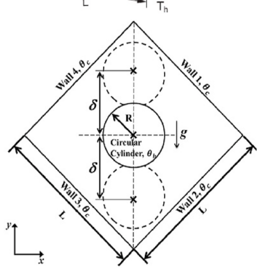
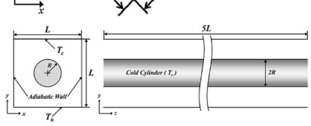
Chiu and Chen [20] numerically investigated the problem of free

**Table 1**
Natural convection inside enclosures with inner bodies and regular fluids (NCIB-RF).

| Authors | Geometry | Solution method and used models | Results and remarks |
|---|---|---|---|
| Burns and Stewart [9] with permission from Elsevier |  | 2D numerical analysis<br>Darcy–Oberbeck–Boussinesq model; permeable outer cylinder; non-primitive variables and finite difference method | A presence of a permeable wall characterizes the energy transport augmentation at the impermeable internal cylinder. |
| Stewart and Burns [10] with permission from Elsevier |  | 2D numerical analysis<br>Darcy–Oberbeck–Boussinesq model; permeable inner cylinder; non-primitive variables and finite difference method | A multicellular hydrodynamic structure appears in the case of equal cylinder temperatures. |
| Sawada et al. [13] with permission from Elsevier |  | Experimental analysis | Natural convection of the magnetic liquid can be managed by the orientation and strength of the magnetic field. |
| Ho et al. [14] with permission from Elsevier |  | 2D numerical analysis;<br>Oberbeck–Boussinesq model; non-primitive variables and finite difference method | An addition of convective boundary conditions at the external surface allows for strengthening the convective flow and heat transfer. |
| Fu et al. [15] with permission from Elsevier |  | 2D numerical analysis;<br>Oberbeck–Boussinesq model; primitive variables and finite element method | The rotation of the cylinder makes an essential contribution to heat transfer enhancement, taking into account the temperature of vertical walls. |
| Lei and Kleinstreuer [16] with permission from Elsevier |  | 3D numerical analysis;<br>Oberbeck–Boussinesq model; primitive variables and finite volume method | The horizontal location of the balloon reflects natural convection domination in the upper half, and heat conduction in the lower half and more than half of the balloon surface can be effectively applied for undesirable tissue ablation. The vertical location illustrates a formation of axisymmetric natural convection with an effective balloon surface area of less than 30%. |
| Chiu and Chen [20] with permission from Elsevier |  | 2D numerical analysis; spherical polar coordinate system; Oberbeck–Boussinesq model; non-primitive variables and finite difference method | Fluid flow and thermal performance depend on the temperature difference and the eccentricity. |

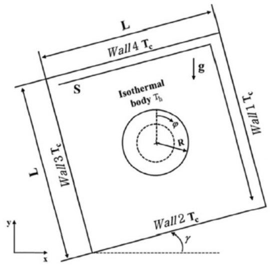
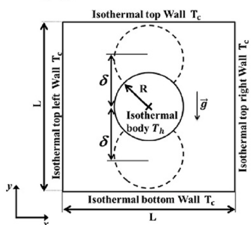
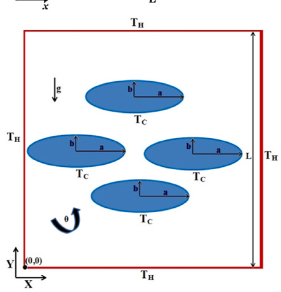
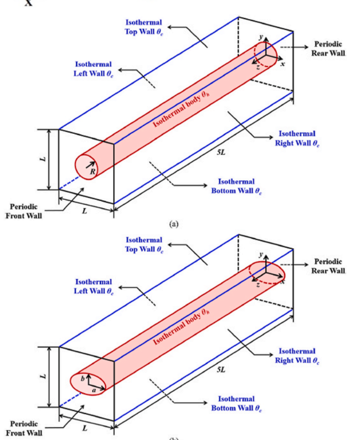
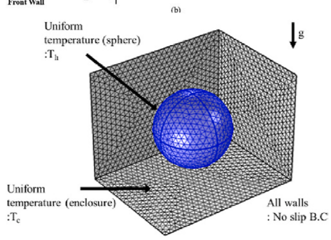
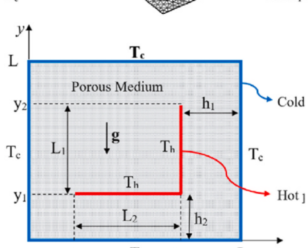
(*continued on next page*)

**Table 1** (*continued*)

| Authors | Geometry | Solution method and used models | Results and remarks |
|---|---|---|---|
| Sun and Emery [22] with permission from Elsevier |  | 2D numerical analysis; Oberbeck–Boussinesq model; primitive variables and finite volume method | In the case of a cavity with an inner body of finite conductivity, energy transport depends on the coupling influence of body conduction, liquid convection, and the body's position within the chamber. |
| Sasaguchi et al. [24] with permission from Elsevier |  | 2D numerical analysis; density-temperature relationship of water; generalized coordinate system; primitive variables and Marker-And-Cell method | The inner cylinder's location and initial temperature allow changing the flow structure and temperature patterns. |
| Ekundayo et al. [25] with permission from Elsevier |  | Experimental analysis | The presence of the horizontal walls caused a reduction in convection for any location of the heated cylinder. In contrast, vertical walls may enhance convection provided that the heater is located approximately halfway up the cavity. |
| *Ha* et al. [26] with permission from Elsevier |  | Numerical study finite volume method | Based on the Rayleigh number and the aspect ratio, substantial variations in heat and flow fields were observed. |
| Butler et al. [48] with permission from Elsevier |  | Experimental study | As the cylinder Rayleigh number increases, they deviate from correlations as the heat transfer rate increases due to the interaction of the cylinder and cavity. |
| Choi et al. [62] with permission from Elsevier |  | Numerical study the immersed boundary method | The Rayleigh number and the placement of the cylinder have an important role in determining the heat transfer characteristics between the enclosure and the cylinder. Buoyancy-induced convection is enhanced by the enclosure's top wall and inner cylinder having gaps. |
| Choi et al. [69] with permission from Elsevier |  | Numerical study the immersed boundary method | The effect of thermal convection on the thermal and flow fields is more concentrated on the lower part of the rectangular channel as the radius of the circular cylinder increases. |

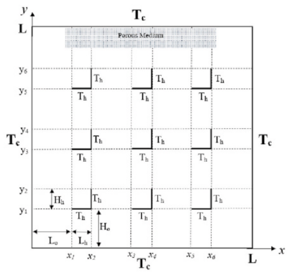
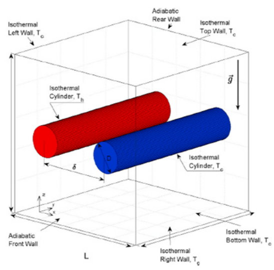
(*continued on next page*)

**Table 1** (*continued*)

| Authors | Geometry | Solution method and used models | Results and remarks |
|---|---|---|---|
| Mun et al. [70] with permission from Elsevier |  | Numerical study immersed boundary method | The unsteady features of the thermal fields and flow occur at low Prandtl numbers and high Rayleigh numbers. |
| Doo et al. [71] with permission from Elsevier |  | Numerical study, the immersed boundary method | The instability of the flow is highly increased at high Rayleigh numbers and low Prandtl numbers Regardless of the location of the cylinder, the system's level of irreversibility stays constant. at the same Prandtl number and Rayleigh number, when the Rayleigh number is low. |
| Kefayati and Tang [88] with permission from Elsevier |  | Numerical study the Bingham model Lattice Boltzmann method (LBM) | The increasing Bingham number results in less heat transport, as seen by the average Nusselt numbers. the study showed the effect of the Rayleigh number, Bingham number, Eckert number, the size and position of the four cold cylinders using LBM |
| Seo et al. [95] with permission from Elsevier |  | Numerical study the immersed boundary method | The variation in radius for the elliptical or circular cylinders highly affects the heat transfer performance. |
| Lee et al. [96] with permission from Elsevier |  | Numerical study, Finite Element Method | The kinematic viscosity and thermal diffusivity of air both increase by increasing temperature, leading to a reduction of the Rayleigh number with the temperature increase. |
| Alomar et al. [99] with permission from Elsevier |  | Numerical study non-Darcian flow and LTNE model Finite volume method. | Nusselt number is highly connected to the modified Rayleigh number The effect of Effective thermal conductivity on the temperature and velocity fields is more than the effect of the scaled heat transfer coefficient of the fluid |

(*continued on next page*)

**Table 1** (*continued*)

| Authors | Geometry | Solution method and used models | Results and remarks |
|---|---|---|---|
| Alomar et al. [104] with permission from Elsevier |  | numerical analysis<br>non-Darcy law combined with LTNE model<br>Darcy-Brinkman-Forchheimer model | Nusselt number is highly connected to the modified Rayleigh number<br>The intensity of the non-equilibrium zone inside the cavity is increased with an increase in modified Rayleigh number<br>The temperature fields are significantly affected by the effective thermal conductivity ratio than the scaled heat transfer coefficient of the fluid |
| Zemach et al. [105] with permission from Elsevier |  | numerical analysis<br>The immersed boundary | The results indicated that the basic instability features of considerably separated thermally driven flows into the confined containers. |

convection in a space between two isothermal spheres using non-primitive variables and finite difference techniques. The authors showed that convective flow and energy transport depend on the temperature difference and eccentricity. Moreover, a positive eccentric shape can augment the energy transport strength. A numerical study was carried out within a vertical square enclosure by Oh et al. [21] to analyze natural convection's steady-state heat transfer and flow properties when a temperature difference occurs around an enclosure. Furthermore, the conductive body produces heat inside the enclosure at the same time. By increasing the temperature-difference ratio, the flow is mainly governed by the temperature gradient around the cavity, which tends to be mostly affected by the temperature gradient induced by the heat source difference. Convective-conductive heat transfer in a square differentially heated cavity with massive walls and internal solid baffles and heaters was numerically investigated by Sun and Emery [22]. Using the finite volume method, they ascertained that in a cavity with a conductive inner body, the energy transport strongly depends on both the combined influence of heat conduction within the body and natural convection within the chamber and the location of the conductive unit within the cavity.

In another notable research, numerical investigations on the free convective heat transfer of micropolar fluids in a horizontal eccentric annulus near maximum density are performed. The results show that the eccentricities and the inversion parameter have a powerful influence on the rate of annulus heat transfer and the flowing fluid structures [23]. Sasaguchi et al. [24] numerically studied the natural convective cooling of water in a thermally-insulated rectangular cavity with an internal cold circular cylinder. Results demonstrated that changes in the inner cylinder location and initial water temperature affect the flow structures and thermal performance. Ekundayo et al. [25] experimentally studied the natural convection of air in a metal box with an inner cylindrical electric heater. The performed analysis showed that a localized chimney effect could be found within the chamber when a local heater placed close to the vertical wall leads to strengthening energy transport. In a square enclosure, the natural convection heat transfer properties of three distinct fluids (water, air, and sodium) were examined [26]. Cesini et al. [27] studied free convection heat transport in a rectangular cavity with a horizontal cylinder using computational and experimental methods. A three-dimensional systematic numerical analysis on the conjugate heat transfer of conduction and natural convection of vertical cubic enclosure in which a centered, cubic, heat-conducting body steadily generates heat [28].

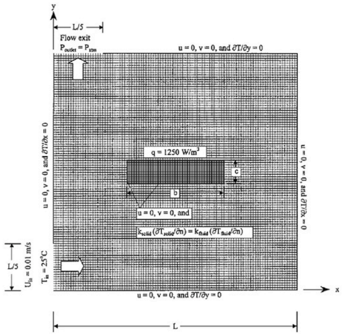
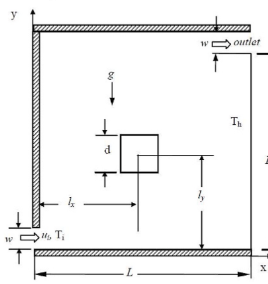
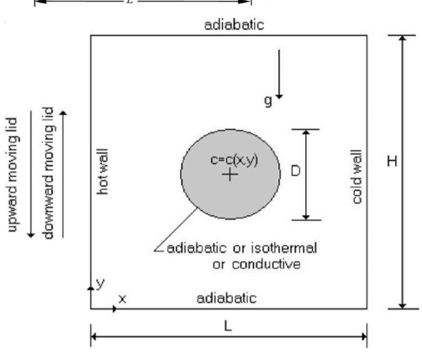
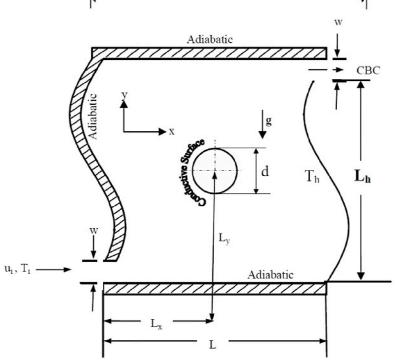
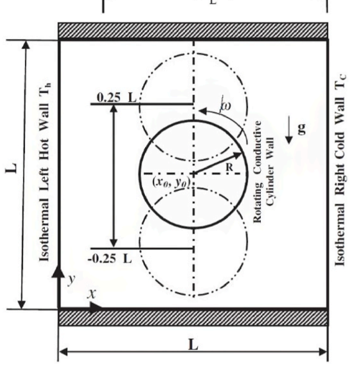
### 2.1.3. *NCIB-RF: (2001–2010)*

In the study of Ha and Jung [29], natural convective heat transfer is numerically handled using the Differential Quadrature (DQ) method. The computational domain in the mentioned work is a horizontal eccentric annulus where the outer and heated inner cylinders have square and circular cross-sections, respectively. The results clearly show that the numerical technique under consideration is an effective way of determining the domain's weak global circulation. As another efficient and accurate method, a two-dimensional Chebyshev spectral collocation method is implemented to solve unsteady natural convection in a square enclosure [30].

In the work of Shu and Zhu [31], natural convection in a concentric annulus is simulated using the DQ technique, in which one of the cylinders is cold whereas the other is hot. The Results referred that flow and thermal field patterns are influenced by the Rayleigh number and the aspect ratio. The thermal and hydrodynamic characteristics of flow were addressed by Tasnim et al. [32] in a cavity with equal horizontal and vertical walls embedded with an isothermal circular cylinder. The obtained results showed satisfactory agreement with those obtained by Cesini et al. [27]. The finite volume technique based on a collocated, non-orthogonal grid was chosen to find out more about the 2D free convection flow and heat transfer in an enclosed hot cylinder by Roychowdhury et al. [33] to investigate the 2D free convective flow and heat transfer in a heated cylinder held in a square cavity. This study provides useful observations regarding the variation of local Nusselt numbers along each wall. Also, there were two methods utilized in the study of Peng [34]: Taylor series expansion and the least-squares based Boltzmann lattice, to numerically analyze the free convective heat transfer between an outer cylindrical square and an inner cylinder that is heated.

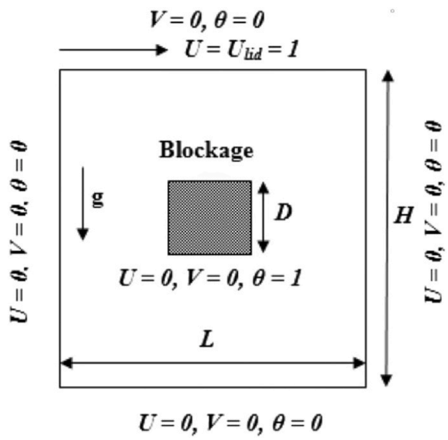
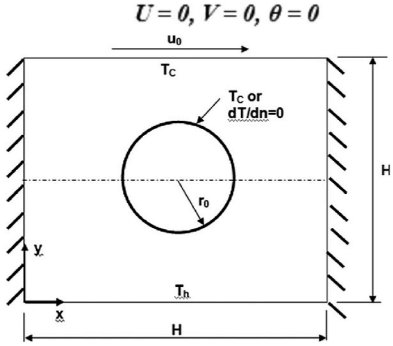
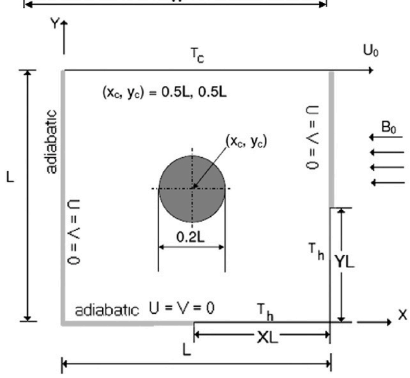
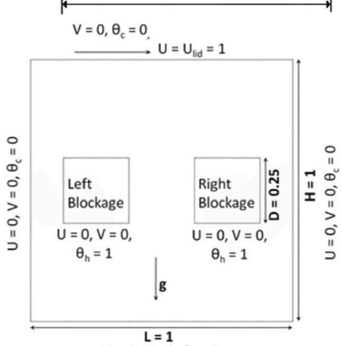
Lee and *Ha* [35] focused on a horizontal fluid layer exposed to natural convection. A body that acts as a conduit was placed at the center of the layer and was affected by heated below and cold above with walls. Their results suggest that, when concerning the flow and heat transfer distributions, the interaction of the surrounding enclosure with the central heat source behaves quite differently. A numerical study of the free convective flow between an exterior square tube and an inside heated circular tuber is investigated by Ding et al. [36] using the local multi-quadrics-based differential quadrature method. In their physical model, the problem of heat transfer and fluid flow was investigated by Lee and *Ha* [37] in a domain where a centered heat-generating conducting body was surrounded by a horizontal layer of fluid-cooled from

**Table 2**
Mixed convection inside enclosures with internal bodies and regular fluids.

| Authors | Geometry | Solution method and used models | Results and remarks |
|---|---|---|---|
| Shuja et al. [113] with permission from Elsevier |  | 2D numerical analysis, control volume approach, Oberbeck–Boussinesq model | Circulation is created inside a solid body's outer surface and front cavity surface by mixing buoyancy-driven and convective flow |
| Rahman et al. [114] with permission from Elsevier |  | 2D numerical analysis, Boussinesq model, Galerkin method of weighted residuals, and finite element method | Increasing the Richardson numbers commonly leads to augmentation of the average Nusselt number. |
| Oztop et al. [115] with permission from Elsevier |  | 2D numerical analysis, Boussinesq model, inner cylinder, finite control volume | Flow domain and temperature pattern are changed from the direction of the moving surface. |
| Hussain and Abd-Amer [116] with permission from Elsevier |  | 2D wavy numerical analysis, Boussinesq model, inner cylinder, finite volume method | The diameter of the embedded solid cylinder and the Richardson and Reynolds numbers have great influences on the average Nusselt numbers of the heated. |
| Hussain and Hussein [117] with permission from Elsevier |  | 2D numerical analysis; Oberbeck–Boussinesq model; non-primitive variables and finite difference method | Increasing the Richardson and Reynolds number leads to the increase of the average Nusselt number rate. |

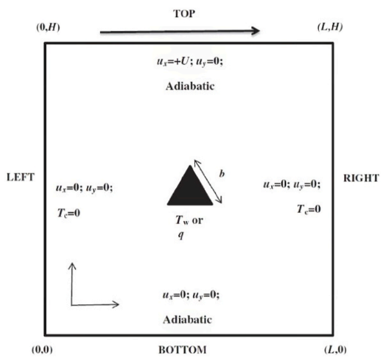
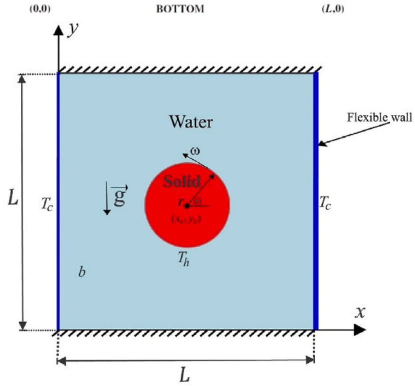
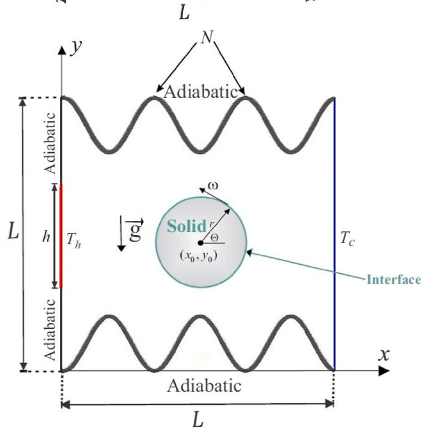
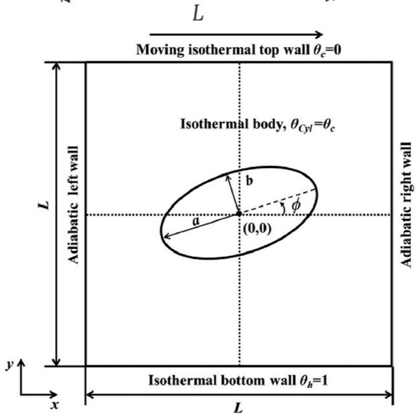
(continued on next page)

**Table 2** (*continued*)

| Authors | Geometry | Solution method and used models | Results and remarks |
|---|---|---|---|
| Islam et al. [118] with permission from Elsevier |   | 2D numerical analysis, Boussinesq model, inner heated square block, finite volume method | For the mixed convection rule, it is found that the average Nusselt number is almost insensitive with the development of the Richardson number to any size of the inner block located anyplace in the square cavity. |
| Khanafer and Aithal [119] with permission from Elsevier |   | 2D numerical analysis with active horizontal walls, finite element method | The mixture of buoyancy-driven and convective flow creates circulation inside a solid body's outer surface and front cavity surface. |
| Ray and Chatterjee [120] with permission from Elsevier |   | 2D numerical analysis, magnetic field, Joule heating, Boussinesq model, commercial CFD package FLUENT | The heat transfer rate reduces drastically as the magnetic field is applied. |
| Morshed et al. [121] with permission from Elsevier |   | 2D numerical analysis, Boussinesq model, inner two heated square blocks, finite volume method | The average heat transfer at the inner block surfaces improved with the rising of Reynolds number on any Richardson number |

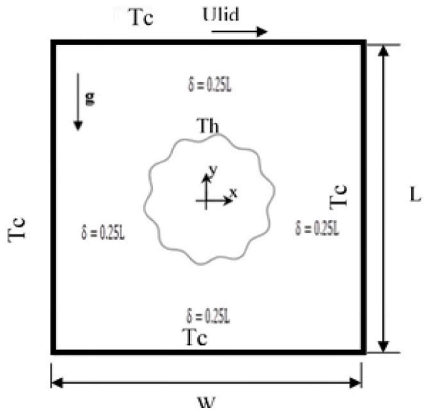
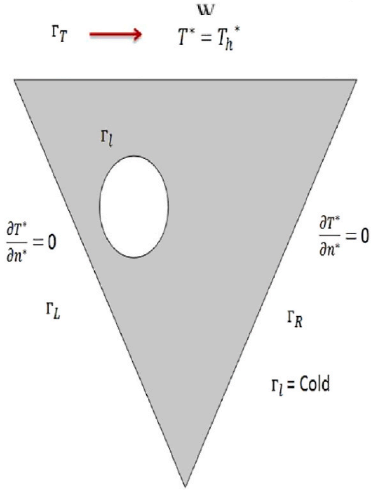
(*continued on next page*)

**Table 2** (*continued*)

| Authors | Geometry | Solution method and used models | Results and remarks |
|---|---|---|---|
| Gangawane and Manikandan [122] with permission from Elsevier |  | 2D numerical analysis, Boussinesq model, finite volume method | A rise in Prandtl number creates a packing of isotherms towards the heated triangular block due to an addition in temperature gradient on the surface of the block. |
| Alsabery et al. [123] with permission from Elsevier |  | Fluid-structure interaction, 2D numerical analysis, heated rotating cylinder; Boussinesq model, finite element method | The highest average heat transfer and global entropy generation rates are observed for the case of counter-clockwise rotation of the circular cylinder and lower rates of the flexible surface deformation. |
| Alsabery et al. [124] with permission from Elsevier |  | 3D numerical analysis, rotating solid cylinder; Boussinesq model, finite-element method | The fluid flow is dominated by managing the circular cylinder rotating velocity of |
| Cho et al. [125] with permission from Elsevier |  | 2D numerical analysis, inclined inner elliptical cylinder; Boussinesq model, artificial neural network | The development of the Richardson number led to the enhancement of buoyancy outcomes. |

(*continued on next page*)

**Table 2** (*continued*)

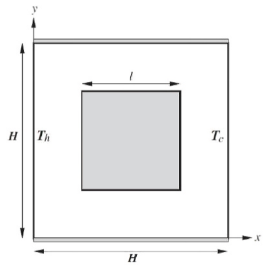
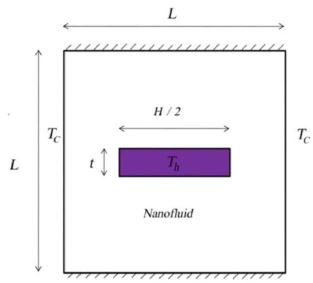
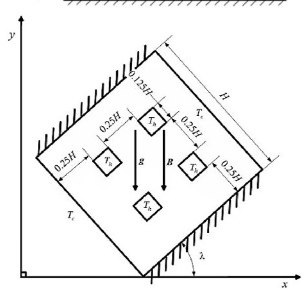
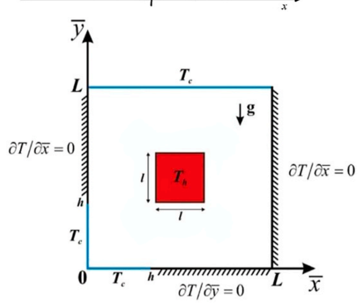
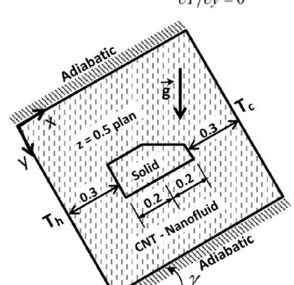
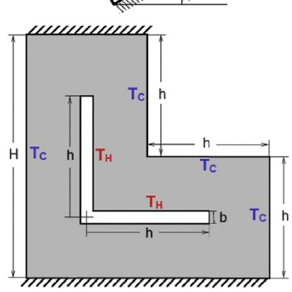
| Authors | Geometry | Solution method and used models | Results and remarks |
| --- | --- | --- | --- |
| Hamza and Mahdi [126] with permission from Elsevier |  | 2D numerical analysis, corrugated hot circular cylinder; Boussinesq model, SIMPLEC algorithm scheme | The flow strength was raised by reductions of Richardson number and the maximum value corresponding with the minimum aspect ratio of the cavity to the circular cylinder. |
| Xiong et al. [127] with permission from Elsevier |  | 2D numerical study, Galerkin Finite Element Method | The Richardson number remains an increasing role regarding the Nusselt number, while the reverse aim does recognize the increase in the Hartmann number. |

above and heated from below. Variations of averaged Nusselt number for various ranges of Rayleigh number were investigated by them. Kumar and Dalal [38] opted for a heated square cylinder that was tilted in an enclosed space. To study the free convection problem. The findings they achieved for the uniform wall heat flux heating are qualitatively different from the results they obtained for the uniform wall temperature heating.

Analysis of free convection in an air filled enclosure with a high-temperature gradient has been performed in Ref. [39]. Different physical mechanisms drove two instability types based on the development of the flow in a square or tall cavity: buoyancy-driven and shear instabilities. Liu et al. [40] considered the natural convective heat transfer in a vertical enclosure subject to periodic temperature boundary conditions imposed at the right sidewall with a conducting body placed at the enclosure center. Results convey that the resonant frequency declines with the rising thermal conductivity ratio and body size. For a heat-conducting cylinder contained inside a differentially heated square cavity at its middle, a numerical analysis of laminar convective flows is conducted out by Jami et al. [41], where the average heat transfer of hot and cold walls, the fields of flow, and temperature are discussed. The work presented by Jami et al. [42] studied the natural convective heat transfer in a laminar regime inside a square cavity embedding a powerful cylinder placed in an arbitrary location. The predicted outcomes indicate that the cylinder position significantly impacts heat transfer. Buoyancy-induced flow regimes are investigated numerically by Angeli et al. [43] for the primary case of a long coaxial square-sectioned cavity embedded with a horizontal cylinder in its center. Also, a two-dimensional (2D) free convection problem in a square enclosure enclosing an adiabatic cylinder at the center has been investigated by Saha et al. [44] using the finite element method. The effect of the size of the heat source on the fluid flow patterns and rate of heat transfer is studied by Kim et al. for natural convection. The temperature difference is a major cause of convection, which is found between an outer cold square enclosure and an inner hot circular cylinder. Such a problem can be solved using the immersed boundary method, which can present a two-dimensional solution for natural unsteady convection.

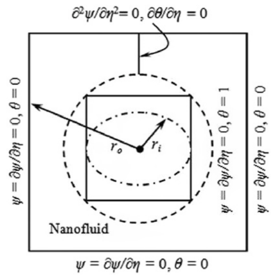
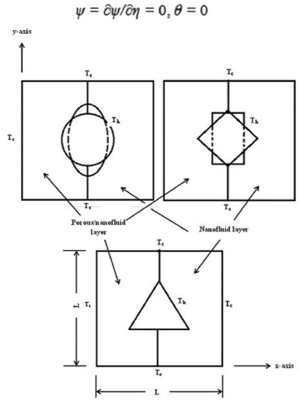
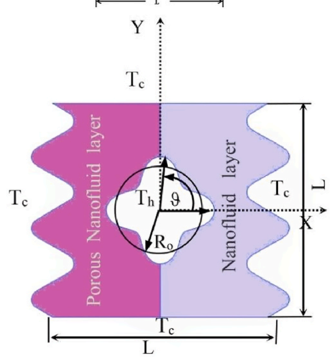
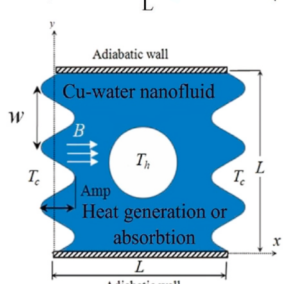
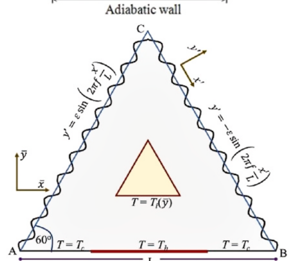
Shi et al. [45] investigated the natural convection in a square enclosure with a rectangular heated cylinder using the lattice Boltzmann technique. The results were obtained for the effect of the cylinder width and the Rayleigh number on the features of fluid flow and the rate of heat transfer. Also, in a similar work, the natural convection originating from the temperature gradient between a hot inner circular cylinder and an outer cold square cylinder is numerically investigated [46]. The derivation of several cells known as Benard cells and the quantitative changes in heat transfer has also been demonstrated. Two-dimensional numerical analysis of steady free convection was accomplished by Hussain and Hussain [47] for a constant heat source applied to the inner circular cylinder in a square enclosure filled with air. It is observed that the flow field does not affect small Rayleigh numbers, while the flow pattern is highly affected in high Rayleigh numbers. Lee et al. [48] carried out numerical calculations for temperature differences between a cold outer square cylinder and a hot inner circular cylinder. Natural convection was analyzed numerically between a heated elliptical inner cylinder and a square outer cylinder [49]. The findings show that the number of Rayleigh and the inner cylinder location is highly dependent on isotherms, streamlines, and the cells' size, number, and configuration.

**Table 3**
Natural convection with inner bodies (single-phase nanofluid).

| Authors | Geometry | Solution method and used models | Results and remarks |
|---|---|---|---|
| Mahmoodi and Sebdani [131] with permission from Elsevier |   | Finite volume method for natural convection fluid flow and heat transfer of Cu– water nanofluid on adiabatic square bodies embedded at the centre of inside a square cavity. | The rate of heat transfer trend is decreasing against the size of the adiabatic square body and nanoparticles volume fraction and, at low Rayleigh numbers, while for high values of Rayleigh numbers, the rate of heat transfer increases. |
| Sheikholeslami and Ganji [135] with permission from Elsevier |   | Lattice Boltzmann method and Koo–Kleinstreuer–Li correlation. Magnetohydrodynamic natural convection flow and heat transfer of CuO–water nanofluid. | The results indicate that the rate of heat transfer and the dimensionless entropy generation number enhances with nanoparticle volume fraction and the Rayleigh number. In contrast, the heat transfer rate declines with the addition of the Hartmann number. |
| Zhang and Che [138] with permission from Elsevier |   | An inclined square enclosure with four heat sources was modeled via MRT thermal lattice Boltzmann simulation to study heat transfer and magnetohydrodynamic (MHD) flow of a nanofluid (Cu–water). | |
| Sheremet et al. [4] with permission from Elsevier |   | Finite volume method and Tiwari and Das nanofluid model. Entropy generation in natural convection. | It is conveyed that the insertion of nanoparticles shows an enhancement in the rate of heat transfer and mitigation of convective flow within the square cavity. In addition, the heat transfer rate declines with the dimensionless inner solid block. At the same time, Dimensionless blocks increase the average Bejan number and overall entropy production. |
| Al-Rashed et al. [142] with permission from Elsevier |   | Control volume finite method and central-difference scheme. Entropy generation and natural convection Within an inclined cubical enclosure filled with nanofluid of CNT-water contained Ahmed's solid body under a temperature gradient. | At a low Rayleigh number, the existence of nanoparticles and their expanded surface area improves the rise in irreversibility. These outcomes grow in entropy due to friction. However, this tendency was switched when the Rayleigh number was raised. The entropy reduces when the inclination angle is boosted up to 90. |
| Rahimi et al. [144] with permission from Elsevier |   | Lattice Boltzmann method and experimental thermo-physical properties. Natural convection heat transfer and fluid flow in a hollow L-shaped cavity are considered and filled with a hybrid nanofluid with $SiO_2$–$TiO_2$/Water-EG. | The Rayleigh number and the solid volume fraction of nanofluids enhance the average Nusselt number. In addition, the generation rate of entropy intensifies at higher Rayleigh numbers and lower solid volume fraction of nanofluid. |

(continued on next page)

**Table 3** (*continued*)

| Authors | Geometry | Solution method and used models | Results and remarks |
|---|---|---|---|
| Roy [146] with permission from Elsevier |  | The free convection of nanofluids in a square cavity with varied internal bodies and distinct kinds of nanoparticles is performed using the finite difference technique. | The results showed that the intensity of streamlines enhances with the boost in the Rayleigh number and the nanoparticle volume fraction. Higher nanoparticle volume fractions show a linear improvement in the Nusselt number inside the inner and outer cylinders, whereas an exponential increase in the Rayleigh number is seen. |
| Hussain and Rahomey [150] with permission from Elsevier |  | Finite element approach and Darcy–Brinkman model. Natural convection and heat transfer in a square cavity with different inner cylinders filled with Ag-nanofluid and superposed porous-nanofluid layers. | The strength of convection heat transfer and fluid flow increases as the thickness of the porous layer thickens, while the intensity of fluid flow and convection heat transfer decreases when the Darcy and Rayleigh numbers, thermal conductivity ratio, and nanoparticle volume percentage are added. Nusselt number values were found to be lower than (60–70%) when the cavity was filled with an overall saturated porous material. |
| Abdulkadhim et al. [152] with permission from Elsevier |  | Finite element scheme and Darcy–Brinkman model. A temperature gradient from the hot inner corrugated cylinder is used statistically to study natural heat transfer convection.—different inner grooved numbers placed in a cooled wavy-walled enclosure filled with Ag nanofluid. | Raising the number of Rayleigh and Darcy numbers improves the strength of the fluid flow and the thickness of the shear layer. The strength of the fluid flow can be conveyed by the high number of the sinusoidal inner surface. Also, increasing the thickness of the porous layer decreases the heat transfer rate. |
| Abdulkadhim et al. [160] with permission from Elsevier |  | Galerkin-weighted residual formulation and finite element method for natural convection of Cu-water nanofluid in a wavy, magnetically-driven cage with a heated, round cylinder within. | At low Rayleigh numbers, the magnetic field had no discernible effect, according to the findings. High Rayleigh numbers boosted heat transmission substantially. |
| Munawar et al. [164] with permission from Elsevier |  | Finite element method for Entropy generation and natural convective hybrid nanofluid flow in a triangle corrugated enclosure with a triangular heater on the inside. | The magnetic field is effective in minimizing the average entropy of the enclosure. Moreover, the large values of the Hartmann number cause heat transfer irreversibility to be augmented. An enclosure with inclined corrugated walls is more effective in minimizing the overall entropy than flat inclined walls, especially at the higher value of the Rayleigh number. |

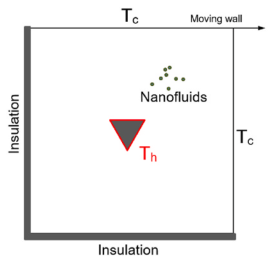
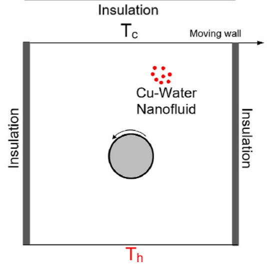
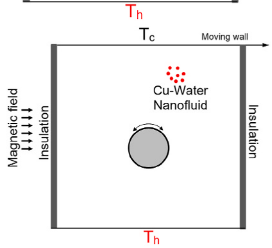
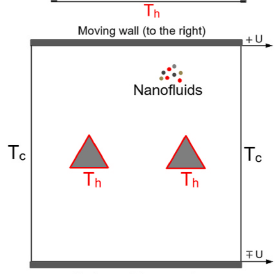
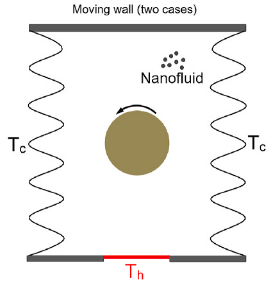
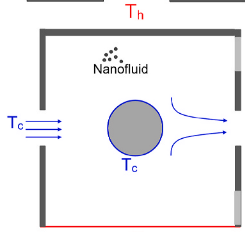
#### 2.1.4. ***NCIB-RF***: (2011–2017)

Yoon et al. [50] investigated the issue of free convection in a rectangular hollow with heated inner cylinders. The cylinders were the same size and had higher temperatures than the enclosure walls. The effects of some governing parameters such as inner cylinders' radius and the flow Reynolds number on the rate of heat transfer and fluid flow were investigated. Their results confirmed that the Nusselt number was dependent on the Rayleigh number and the radius of the inner cylinders.

Park et al. [51] investigated the temperature difference induced by natural convection in a cold square enclosure containing hot and cold circular cylinders. The study provided a comprehensive results of Nusselt number distribution, isotherms and streamline countours. An immersed-boundary method was implemented to numerically investigate the mixed and natural convection within domains with rotating and stationary complex geometry [52]. A detailed study of free convection over a heated elliptical cylinder at the center of a cold square cavity is conducted. The article investigated the relationships between the Rayleigh number, cavity aspect ratios, and the axis ratio [53].

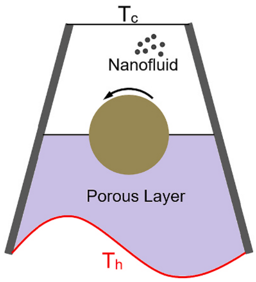
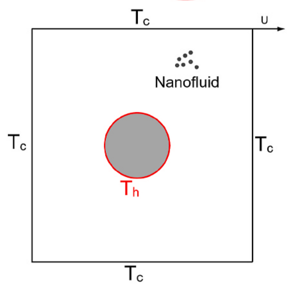
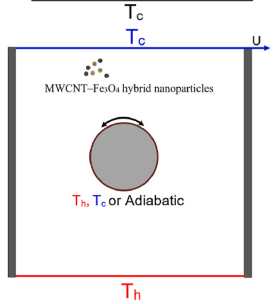
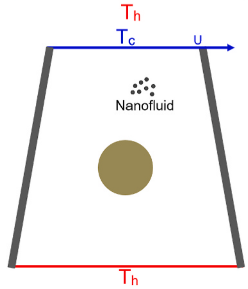
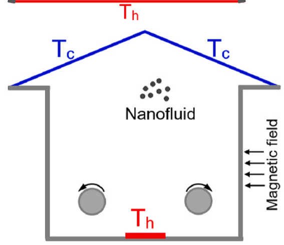
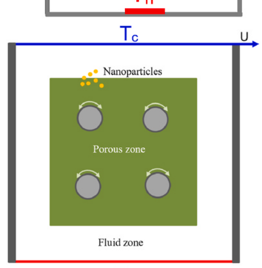
The study of Kang et al. [54] dealt with numerical fluid flow and heat transfer investigations on the location of inner heated cylinders in cooled enclosures on the Rayleigh number equal to $10^7$. In the work of Butler et al. [55], the free convective heat transfer from a heat-generating horizontal cylinder enclosed in a square cavity was studied. Results show that at the low Rayleigh numbers, the available

**Table 4**
Mixed convection with inner bodies (single-phase nanofluid).

| Authors | Geometry | Solution method and used models | Results and remarks |
|---|---|---|---|
| Kalteh et al. [165] with permission from Elsevier |  | Finite difference method for stream-function-vorticity formulated equations. Brinkman model for viscosity, while the Brownian motion is considered for the thermal conductivity. | All nanoparticles examined showed the positive role of nanofluid. The Ag nanoparticles gave the highest Nusselt number, while $TiO_2$ nanoparticles were the worst among them. Smaller size nanoparticles augment Nusselt numbers better than big ones. |
| Chatterjee et al. [166] with permission from Elsevier |  | CFD FLUENT package implementing finite volume method. The Brinkman model for viscosity and Maxell-Gannet for thermal conductivity. | Cu–H2O nanofluid increases the Nusselt number and reduces the drag coefficient. The rotation of the adiabatic centered cylinder plays an adverse action on the Nusselt number. Moving the top wall and the rotating cylinder in opposite directions raises the drag coefficient. |
| Selimefendigil and Oztop [167] with permission from Elsevier |  | Finite element method. Brinkman model for viscosity and Maxell-Gannet for the thermal conductivity. | Nanofluid enhances the Nusselt number. Nusselt number is enhanced when the inner adiabatic cylinder rotates clockwise, i.e., in assistance mode with the top moving wall. Although the magnetic field retards the Nusselt number, it can be considered as a parameter to control the heat transfer inside the cavity. |
| Boulahia et al. [168] with permission from Elsevier |  | Finite volume method. Corcione models for viscosity and thermal conductivity implementing the Brownian motion. | Although there are no notable deviations among the Nusselt numbers of different nanoparticles, copper gave the highest Nusselt number while titanium oxide was the worst. The size of nanoparticles plays a notable role in enhancing the Nusselt number, while the direction of the bottom wall does not. |
| Alsabery et al. [169] with permission from Elsevier |  | Finite element method. Corcione models for viscosity and thermal conductivity implementing the Brownian motion. | The rotation of the cylinder is useless for the high Rayleigh number. The waviness of the vertical walls inferred the friction irreversibility greater than the thermal one. The volume fraction of the alumina raises the Nusselt number with an ignorable impact on the entropy generation. |
| Boulahia et al. [170] with permission from Elsevier |  | Finite volume method. Corcione models for viscosity and thermal conductivity implementing the Brownian motion. | When the outlet port is located in the upper or middle of the right wall, the role of nanoparticles scales down for weak and moderate natural convection. The lower the outlet port is, the better the t heat transfer |

(*continued on next page*)

**Table 4** (*continued*)

| Authors | Geometry | Solution method and used models | Results and remarks |
|---|---|---|---|
| Hussein et al. [171] |  | COMSOL Multiphysics to solve the governing equations. Maxwell-Garnett and Brinkman formulas to predict the thermal conductivity and viscosity, respectively. | Nusselt number increases with the nanoparticles volume fraction, Darcy number, and rotational speed of the cylinder. The effect of the rotating cylinder was much evident in the regular layer. The height of the porous layer weakens the convective heat transfer |
| Thakur et al. [172] with permission from AIP Publishing |  | Galerkin finite element method. Maxwell-Garnett and Brinkman formulas to predict the thermal conductivity and viscosity, respectively | The augmentation of Nusselt number due to the Reynolds and Grashof numbers is much higher than that due to the addition of nanoparticles. The role of the nanoparticles in enhancing the Nusselt number decreases with higher values of Reynolds and Grashof numbers. |
| Jamal and Hussain [173] |  | Galerkin finite element method. Maxwell-Garnett and Brinkman together with the general mixing rules formulas, were used to predict the nanofluid properties. | No distinct fashion regarding the hybrid nanofluid compared with regular nanofluids. When the cylinder rotates incompatible with the top wall movement, the Nusselt number is maximum. |
| Re-plotted from Ishak et al. [174] |  | Galerkin finite element method. Corcione model for viscosity and thermal conductivity. | The alumina nanoparticles raise the Nusselt number and the rate at which entropy is generated. In terms of heat transmission and entropy, the inner cylinder's size and placement were the most important factors. |
| Re-plotted from Ali et al. [175] |  | Finite element method implemented in COMSOL Multiphysics. The thermal conductivity takes into account the high-speed motions of nanoparticles which is formulated by adding a dynamic term to the primitive Maxwell-Garnett model. The viscosity was modeled based on a polynomial relation (Pak and Cho). | Increasing the rotating speed and placing the heat source in the corner of the enclosure result in a considerable increase in heat transmission. High percentage augmentation of Nusselt number when raising the volume fraction of 5%. The magnetic field suppresses fluid circulation and heat transfer. |
| Re-plotted from Shirani and Toghraie [176] |  | Finite volume method. Maxwell-Garnett for thermal conductivity and Pak and Cho for viscosity. | Distinct results were found, where the Nusselt number elevates with decreasing Darcy and Richardson numbers. The porosity has no decisive effect on the Nusselt number, and the contribution of the harmonic rotation of the four cylinders becomes more perceptible at higher permeability. The role of the nanoparticles was positive, where under certain circumstances, setting the volume fraction at 3%, the Nusselt number rises to 30%. |

literature correlations can well predict the heat transfer from the cylinder. However, as the Rayleigh number increases, where the overall heat transfer from the cylinder increases, some deviations are seen. Two-dimensional numerical simulations are performed by Lee et al. [56] for the natural convection case in a cold enclosure containing a hot inner cylinder at the center. The results show that variations in the dimensions of the bottom-wall local heating zone can lead to minor modifications to the thermal and flow field at Ra = $10^3$ and $10^4$. A numerical study investigated a temperature difference induced by natural convection between two hot inner circular cylinders and a cold outer square enclosure. The effects of the two cylinders located on the heat transfer and laminar fluid flow are investigated in Ref. [57]. Park et al. [58] revealed the natural convection caused by a temperature differential between a hot internal circular cylinder and a cold outer inclined square enclosure. The findings demonstrated that the simultaneous impacts of convection and the relative distance between the walls of the enclosure and the cylinder determine the distribution of streamlines, isotherms, and local and surface averaged Nusselt numbers.

In the work of Liao and Lin [59], numerical immersed boundary investigations are performed for free convection within curved geometry domains. Free convection caused by a temperature difference between an exterior cold square enclosure and two heated interior circular cylinders was explored by Park [60]. Also, compared with a single-cylinder example, results for two-cylinder and single-cylinder cases are compared to see how the interaction with the two heated inner cylinders affects the overall results. Yoon et al. [61] numerically solved two-dimensional natural convection of two circular cylinders located in a cooled square enclosure, with different isothermal conditions at various Rayleigh numbers set in the range ($10^3$-$10^5$). The numerical simulations of Choi et al. [62] are used by the immersed boundary method (IBM). On the basis of a Prandtl number (0.7) and a rhombic enclosure, results for the effects of natural convection on temperature distributions are presented. In a porous enclosure containing heat sources at high temperatures mounted on top and bottom walls, thermal characteristics and fluid flow patterns associated with free convective heat transfer are examined, where the results show the generated high thermal gradient in the vicinity of heat sources leads to the formation of irreversibility that maximizes the entropy generation [63]. A new flexible forcing immersed boundary (IB)–lattice Boltzmann method (LBM) was implemented to investigate the natural convection process in a hot eccentric square cylinder with an annulus of a cold square enclosure [64].

Dash and Lee [65] extended the previous numerical study [64] and investigated diagonal and horizontal eccentric displacement in a square cylinder for the problem of buoyancy-induced convective heat transfer and fluid flow. They gave a comprehensive analysis of the distribution of the Nusselt number, streamlines, and isotherms as functions of the Rayleigh number. An inquiry of the effects of Prandtl and Rayleigh numbers is performed on the results of a buoyancy-induced flow for heated cylinders enclosed within square domains [66]. Choi et al. [67] presented the numerical simulations using the three-dimensional immersed boundary method to study the natural convective heat transfers in a cubical enclosure containing a circular cylinder. The effects of the location of the inner circular cylinder on the buoyancy-induced convective heat transfer and fluid flow in the cubical enclosure were investigated. The numerical study of Park et al. [68] investigated a temperature difference that induced natural convection heat transfer between four hot inner circular cylinders and a cold outer square enclosure in a rectangular array. A 2D immersed boundary method was implemented for the solution of natural convection in the enclosure containing four cylinders. Depending on 3D simulations, numerical natural convection results are presented in a rectangular channel with an inner cold circular cylinder [69].

The temperature difference induced natural convection heat transfer investigations were conducted to numerically simulate the Two-dimensional domain between a hot inner circular cylinder and the cold walls of the square enclosure. Mun et al. [70] used an efficient and accurate numerical immersed boundary method to obtain the detailed analysis results; the distribution of Nusselt numbers, isotherms, and streamlines are all included in this study. While in the study of Doo et al. [71], Convection cell shapes and heat transport features were examined. The natural convection phenomena were numerically simulated using the immersed boundary method (IBM) around a cold cubic enclosure with a hot inner circular cylinder to capture the inner cylinder virtual wall boundary, using the finite volume method (FVM) [72,73]. The variational multiscale element-free Galerkin method (VMEFG) was adopted by Zhang et al. [74]. They numerically investigated a steady-state study for free convection in an outside square cage with an interior elliptic cylinder that is hotter than the surrounding environment. Inverse conjugate free convective heat transfer problem with several unidentified heat sources is examined by Zhang et al. [75] based on temperature sensing inside the enclosure using the conjugate gradient method. The inverse solutions obtained with simulated temperature measurements could be gained by the proposed inverse problem approach for the unknown heat flux functions. In a vertical container, Hu et al. [76] quantitatively analyzed the influence of solid impediments on laminar natural convection. To find optimal arrangements for heat transfer improvement in the cooling of electronic units, where the results demonstrated that there is a rising correlation between Rayleigh numbers and the average Nusselt number and solid-to-fluid thermal conductivity ratio and a decreasing function of solid-to-fluid volume ratio. Results indicate that the existence of solid impediments inside a fluid that is naturally convecting causes flow interference, which can have a dominant impact on the rate of heat transfer and flow patterns. The results proved that horizontal interference predominates concerning vertical interference in shallow enclosures [77]. Souayeh et al. [78] numerically examined a temperature difference 3D free convection of air between a hot inner cylinder and a cold outer cubic enclosure, where the results indicated that at Rayleigh number equal to $10^6$ and an inclination of 90°, the optimal average heat transfer is readily available er rate can be obtained for both cases of lateral walls of the cubical enclosure and the inner cylinder. Using the Lattice Boltzmann Method (LBM), two-dimensional magneto-hydrodynamics (MHD) stable laminar free convection flow in a square enclosure with an electrically conducting fluid was examined computationally by Hussein et al. [79], which the Prandtl, Rayleigh, and Hartmann numbers have an essential role on the thermal and flow features.

Seo et al. [80,81] described a comprehensive parametric investigation on rectangular arrayed cylinders where the effect of the cylinder's location on the flow characteristics in various flows was examined based on the immersed boundary method. The review of Das et al. [82] summarized the free convective heat transfer studies in curved and wavy trapezoidal, triangular, parallelogrammic cavities filled with fluid or porous medium or nano-fluids. Jelita et al. [83] studied the free convective heat transfer between a hot inner elliptical cylinder and a cold outer square enclosure where convective flow depends on the Rayleigh number, elliptical shape, and orientation. The study of Xu et al. [84] adopted LBM to investigate the double-diffusive natural convection around a heated cylinder in an enclosure filled with a porous medium.

#### *2.1.5. **NCIB-RF:** (2018–2022)*

It was noted that the number of researches increased during this period. Thus, it deserves a separated subsection. In the work of Kefayati and Tang [85], the Finite Difference Lattice Boltzmann Method (FDLBM) was used to investigate the double-diffusive natural convection, viscous dissipation, Dufour and Soret effects in a heated cylinder with non-Newtonian fluid. Free convection in a heated inclined cavity filled with viscoelastic fluids and with single and multiple inner cold circular/elliptical cylinders has been simulated using the Lattice Boltzmann Method (LBM), where the results indicated that increasing the Rayleigh number enhances the heat transfer, with a decrease in the dimensions of the unyielded zones [86–88]. Kefayati and Tang [89] examined the thermosolutal entropy generation and free convection in a heated cavity filled with a non-Newtonian Carreau fluid with two inner cold cylinders by LBM. The results indicated that heat transfer for many examined parameters is enhanced as the Rayleigh number increases, for example, the Buoyancy ratio, Hartmann number, and power-law indexes. Aberuee et al. [90] investigated the problem of natural convection heat transfer inside an industrial oven with an internal plate. The results proved that the total rate of heat transferred by a cavity with an internal isolated plate is always lower than that of an empty cavity. The case of two-dimensional natural convection was numerically investigated by Cho et al. [91] in a square cavity consisting of a vertical array of two circular and elliptical cylinders at a specific range of Rayleigh numbers. Mun et al. [92] considered two-dimensional numerical simulations and free convection in a four-cylinder diamond-shaped enclosure. The authors of the study of Seo et al. [93] are concerned with the problem of free convective heat transfer inside an industrial oven with an internal plate.

Zhao et al. [94] used the lattice Boltzmann model to simulate the conjugate natural convection in a rectangle containing a cylinder to investigate the effect of heat capacity. It has been discovered the heat transfer rate obtained for different wall thicknesses is usually smaller than that for the zero-wall thickness. At the same time, this difference is not apparent for a larger thermal conductivity ratio. A 3D numerical simulation study [95] was conducted to analyze the natural convection in a long, cold, rectangular enclosure with an inner hot circular cylinder. The virtual cylinder wall boundary was captured based on the finite volume method (FVM) using IBM, where the heat transfer characteristics were influenced by the radius of the cylinders. Lee et al. [96] investigated internal 3D natural convection heat transfer between a spherical surfaces housed within a cuboidal enclosure. Obtained results revealed the existence of a critical Rayleigh number beyond which the Nusselt number reduces as the temperature difference augments. The review of Pandey et al. [97] presented a comprehensive summary of experimental and numerical surveys and examinations related to laminar natural convection inside enclosures with and without inner bodies. Cho et al. [98] numerically focused on studying the 2D simulations of the free convection in a cold square enclosure with a vertical array of two hot elliptical cylinders at different *Ra* ranges ($10^4$-$10^6$). IBM, based on the finite volume method (FVM), was used to capture the virtual wall boundary of the cylinders.

Authors in the work of Alomar et al. [99] studied the convective heat transfer over two perpendicular plates embedded inside a porous square cavity using non-Darcian flow and Local Thermal Non-Equilibrium (LTNE) assumptions. The results showed that the average Nusselt number is robust to the modified Rayleigh number and inertia parameter. The research of Dutta et al. [100] reconstructed a developed compact finite difference approach for the Navier-Stokes (N–S) equations in the biharmonic and combined it with a high order compact (HOC) scheme for the energy equation to compute the flow around the heated circular and diamond cylinders inside a square enclosure. The study results on nonuniform grids without transformation have an excellent match with available numerical results for both adiabatic and isothermal partitions of the square. Hedia et al. [101] numerically simulated the free convection phenomena between a hot inner body and its outer enclosure. The results found that the thermal and flow fields eventually reach a steady state for Rayleigh numbers ranging from $10^4$ to $10^7$. Boukendil et al. [102] examined the hydrodynamic and the thermal behavior of the fluid, and the radiative and the convective heat transfer are investigated for Rayleigh numbers ranging from $10^3$ to $10^7$, where the obtained results indicate that as the Rayleigh number in the absence of the surface radiation is increased, the convection is strengthened above the heating element. At the same time, conduction persists below the passive portion of the circular body. The effects of exponentially temperature-dependent viscosity on free convection in a porous cavity with a circular cylinder are investigated numerically via the lattice Boltzmann (LB) method based on local thermal non-equilibrium (LTNE) conditions, indicating that by increasing the LTNE parameters such as the thermal conductivity ratio of solid-to-fluid and the coefficient of the interphase heat transfer, the solid heat transfer rate increases dramatically. At the same time, few changes are found in the heat transfer rate of fluid [103]. Alomar et al. [104] presented the characteristics of natural convection induced by a bank of orthogonal heated plates instilled inside a porous cavity using the assumption of local thermal non-equilibrium (LTNE) and non-Darcian conditions. Numerical solutions confirmed that the average Nusselt number (*Nu*) robustly depends on thermal conductivity ratio, inertia parameter, and modified Rayleigh number in comparison with scaled heat transfer coefficient, and the results show that the solid Nusselt number is lower than fluid Nusselt number ($Nu_f$) for an exact value of the inertia parameter. Natural convection flow in a cold cubic cavity was studied by Zemach et al. [105] for its instability features considering a tandem of horizontally aligned cold and hot cylinders. Results indicated that the spatio-temporal symmetry preservation in a moderately supercritical flow is affected by boundary distance and object orientation in containers with substantially separated thermally driven flows, and this leads to fundamental instability.

Unsteady natural convective heat transfer and flow in a heated circular cylinder enclosed in a square enclosure with various vertical locations were numerically analyzed. A comprehensive study has been presented on time-periodical natural convection induced by the sinusoidal temperature of the inner circular cylinder with different vertical locations [106]. A numerical study has been presented to analyze the characteristics of natural convection induced by a bank of orthogonal heated plates instilled inside a porous cavity using the assumption of local thermal non-equilibrium (LTNE) and non-Darcian conditions. The numerical solutions demonstrated that the average Nusselt number robustly depends on thermal conductivity ratio, inertia parameter, and modified Rayleigh number compared to the scaled heat transfer coefficient [104]. Vijaybabu [107], using the LB method, analyzed the significance of porous circular cylinder and magnetic field on the double-diffusive free convective heat transfer and the generation of entropy in an enclosure. The results explained that the improvement in thermal and concentration gradients produced at higher permeability enhances the strength of buoyancy force. Thus, an augmentation in flow-field intensity occurs around the porous cylinder. Some other enclosures contain blades [108,109] or cylinders [110] which are essential for mixture applications but have not yet been investigated for nanofluids.

### 2.2. Mixed convection with internal bodies-regular fluid (MCIB-RF)

The mixed convection flows or a combination of forced and free convection flows that occur in many industrial and technological applications in nature, e.g., fan cooling of electronic devices, wind currents flowing over solar receivers, cooling of nuclear reactors in an emergency shutdown, heat exchangers located in a low-velocity environment, oceanic and atmospheric flows, and so on. In mixed convection, the forced and the natural convection effects are of equivalent magnitude. The lid-driven cavity flow is a standard benchmark problem for investigating several interesting fluid flow aspects. A significant number of research have concentrated on this problem, and an excellent review theme was described by Shankar and Deshpande [111]. We can characterize the mixed convection cases using the Grashof number (*Gr*) and the Reynolds number (*Re*) for natural and forced convection, respectively. The Richardson number expresses the qualified impact of buoyancy on mixed convection:

$$Ri = \frac{Gr}{Re^2}$$

A small value of Richardson numbers describes forced-convection-dominated flow. Richardson numbers higher than *Ri*, approximately 16, describe the flow case as simple natural convection, and the forced convection effects can be considered negligible [112,113]. Similar to natural convection, the originating mechanism of mixed convection flow depends significantly on heat transfer (as buoyancy), and turbulence effects play a vital role.

Rahman et al. [114] studied numerically with the Galerkin weighted residuals finite element, the mixed convection flow and heat transfer of a heat-conducting square cylinder is located at various points in the square hollow. They considered various values of Richardson numbers and the inner cylinder position. They found that in the forced convection-dominated area, the highest value of the average Nusselt number occurs when the cylinder is in the vicinity of the top wall along the mid-vertical plane. However, this condition in the natural convection-dominated scopes is met when the cylinder is moved closer along the mid-horizontal plane to the left vertical wall. In a lid-driven cavity housing a circular cylinder body, Oztop et al. [115] described the situation of fluid flow and heat transmission owing to mixed convection. They used the finite control volume numerically for three temperature boundary conditions employed toward the inner solid cylinder as 1) adiabatic, 2) isothermal or 3) conductive condition. They found that the thermal conductivity traces are negligible concerning the small diameter of the circular solid body.

Hussain and Abd-Amer [116] considered the mixed convection flow and heat transfer of air located inside a wavy hollow with a horizontal circular heat-conducting cylinder they reported that the average Nusselt numbers on the heated surface are greatly influenced by the Richardson and Reynolds numbers and the size of the embedded solid cylinder. Hussain and Hussein [117] considered the laminar steady mixed convection problem in a two-dimensional square cavity by employing the finite volume method. The enclosure is filled with air, and the left surface is dominated by an isothermal temperature higher than the right surface. The top and bottom cavity surfaces are held adiabatic. The outcomes demonstrate that the improvement in the Richardson and Reynolds numbers holds an important function in the flow and temperature ranges. The rotating cylinder positions have a significant influence on improving convection heat transfer within the square cavity. Islam et al. [118] reported the case of the mixed convection flow and heat transfer inside the upper surface lid-driven square cavity with an isothermally heated inner block using the finite volume method. Some parameters were used in the investigation, such as; the inner block ratio and location, Reynolds number, Grashof number, and Richardson number. They found that for the mixed convection rule, it is determined that the average Nusselt number does not change significantly with the development of the Richardson number to any size of the inner block located anywhere in the square cavity.

Khanafer and Aithal [119] reported the heat transfer and laminar mixed convection flow features inside a lid-driven cavity in an oscillating cylindrical shape using the finite element technique. The top cold surface moves in the right direction with a constant velocity while the bottom surface remains maintained at a hot temperature. The vertical surfaces of the cavity are examined adiabatic. They observed that the average heat transfer grows with an addition in the Richardson number for all the examined non-dimensional cylinder sizes. Ray and Chatterjee [120] reported the magnetic field and Joule heating impact of a lid-driven cavity on the combined convection and heat transfer having heat conducting a solid inner object and corner heater. They found that the form of the inner object produces a significant impact on thermal transport. Hence, solid objects with high thermal conductivity are employed to develop a conjugate heat transfer tool and utilize a magnetic field to control heat transfer inside the cavity region. It is realized by Morshed et al. [121] that the average heat transfer at the inner block's surfaces elevates with the rising of the Reynolds number on any Richardson number inside a square cavity having two inner heated square blocks. Three arrangements of the two square blocks had resulted in three correlations to the Nusselt number.

In another work, Gangawane and Manikandan [122] considered the case of mixed convection and heat transfer features within a lid-driven cavity, including a heated triangular block. Alsabery et al. [123] reported the entropy generation and mixed convection flow with fluid-structure interaction analysis within a square cavity with a flexible horizontal surface and heated inner rotating cylinder using the finite element method. They observed that the highest average heat transfer values are obtained for counter-clockwise rotation of the circular cylinder and lower rates of flexible surface deformation.

Other work by Alsabery et al. [124] indicated that the fluid flow could be dominated by managing the rotating velocity of the circular cylinder. Cho et al. [125] used an artificial neural network to predict the performance of heat transfer, and mixed convection flows into a lid-driven square cavity with an elliptical tilted cylinder. They showed that the development of the Richardson number led to the enhancement of the buoyancy outcomes. Hamza and Mahdi [126] examined the study of heat transfer aspect and mixed convection flow inside a square cavity with a modified aspect ratio that has a grooved heated circular body. Different aspect ratios of the enclosure have changed (0.75, 1, and 1.5). They found that the local heat transfer becomes a periodic form throughout the hot inner cylinder to all aspect ratios and begins to drop about Richardson number displays below unity. Xiong et al. [127] explored the numerical simulation of the heat transfer, and MHD mixed convection flows within a lid-driven triangular cavity by various obstacle forms. They noticed that the Richardson number remains an increasing role regarding the Nusselt number, while the reverse aim does recognize the increase in the Hartmann number.

## 3. Convective heat transfer in enclosures with inner bodies (single-phase model)

In this section, we will review the studies of convective heat transfer in enclosures involving inner bodies while the annulus space is filled with nanofluids and simulated using a single-phase flow. Natural and mixed convection studies are grouped in the following two subsections.

### *3.1. Natural convection with inner bodies (single-phase nanofluid)*

Thermal convection is a method of transferring energy that combines conduction, energy collection, and medium movement. Improving and/or controlling heat transport phenomena inside thermal systems is crucial due to industrial applications, including device cooling, heat exchangers, built-in storage, and power generation. As a result, researchers have suggested several successful approaches, including changing the thermophysical characteristics of the working fluids (for example, employing nanofluids), modifying the system geometry within which the phenomena occur, or employing internal/external stimuli such as magnetic field, internal heating or absorbing, and external heating or cooling, etc. Several researches on nanofluids investigated the synthesis and stability of nanofluids are the prerequisites for using them as a heat transfer fluid [128,129].

In the work of Yu et al. [130], the heat transfer was numerically studied by unsteady natural convection in a heated inner horizontal circular cylinder enclosed via a coaxial triangular box for a wide range of Grashof numbers, aspect ratios, and tilt angles of a triangular cylinder. The results showed that different stages during the flow evolution pathway were determined by the developments of the mean Nusselt number on the inner circular wall. It has been studied numerically the issue of free convection fluid flow and heat transmission of Cu–water nanofluid inserted in a square cavity. The obtained results convey that the average Nusselt number rises as the volume fraction of the nanoparticles increases, Mahmoodi and Sebdani [131].

A numerical study is carried out concerning the natural convective heat transfer of a nanofluid in a 2D square enclosure enclosing several pairs of heater and coolers (HACs) by Garoosi et al. [132]. However, the results indicate that HACs spatial distribution has a dominant influence on the rate of heat transfer. The augmentation of natural convective heat transfer in nanofluids from a horizontal square cylinder located in a square cavity is numerically investigated. The obtained results demonstrated that raising the volume percentage of nanoparticles at all Rayleigh numbers increases the average Nusselt number, Kahwaji and Ali [133]. The work of Abdallaoui et al. [134] calculated the natural convection around a decentred triangle cylinder in a square cylinder using the lattice-Boltzmann technique. The Lattice Boltzmann Method is used by Sheikholeslami and Ganji [135] to investigate the natural convection flow of CuO–water nanofluid in a square cavity with a rectangle heated body. Results obtained showed that heat transfer and fluid flow characteristics are highly affected by the position of the heating cylinder, and the average Nusselt number is positively influenced by the increase of nanoparticle volume fraction for all considered heating block positions.

Boulahia and Sehaqui [136] used numerical tools to focus on the problem of free convection in a square enclosure embedding a centrally-placed heated block that was filled with Cu–water nanofluid, where the results revealed the increase in the surface area of the heated body had enhanced the heat transfer. Nanofluid flow and heat transfer rates are examined numerically in a heated circular and elliptical cylinder domain in the confines of a chilled cubic enclosure by Ravnik and Škerget [137]. Results show that for the conduction-dominated flow regime, the highest heat transfer enhancement occurs where thermal characteristics of nanofluids have a significant impact.

Zhang and Che [138] introduced a simulation of Cu–water nanofluids under the magneto-hydrodynamic and heat transfer effects in a four-heat-source inclined cavity was performed using a 2D double multiple-relaxation-time (MRT) thermal lattice Boltzmann model. Adding Cu had a higher effect on flow fields than temperature patterns, according to the findings of the study. Natural convective heat transfer of Cu-water nanofluid was numerically investigated in a square enclosure having a cold obstacle. It was demonstrated that the heat transfer rate inside the enclosure stimulates increasing the volume fraction of nanoparticles, the cold block height, and the Rayleigh number, Boulahia et al. [139]. The solid isothermal partition insertion effects in a nanofluid-filled enclosure chilled by an isothermal cooler at the corner were investigated by Sheremet et al. [140] via computational study. Obtained results show that the adoption of nanoparticles increases the heat transfer and attenuates the convective flow inside the enclosure. A vorticity-vector potential technique was used to conduct a 3D numerical survey of the natural convective heat transport in an inclined cubical enclosure filled with CNT-water nanofluid. The results clarified that heat transfer can be enhanced using the CNT particles in all of the considered cases, Al-Rashed et al. [141]. Similarly, the 3D vorticity-vector potential formalism was adopted to evaluate the entropy generation inside an inclined cubical differentially heated enclosure filled with CNT-water nanofluid using FVM. The obtained results convey that the buoyancy-driven Raleigh number augmentation leads to the increase in entropy generation, as explained by Al-Rashed et al. [142]. The natural convective heat transfer was simulated using the lattice Boltzmann method in enclosures filled with DWCNTs-water nanofluid and with active internal rigid bodies by Rahimi et al. [1], where isothermal lines and streamlines are significantly influenced by the arrangement of refrigerant bodies available.

The study of Dogonchi et al. [143] aimed to survey the free convection in a local triangle heater's upper part of a circular horizontal cylinder using the control volume finite element method (CVFEM). Copper-water nanofluid was selected as the working fluid, and a uniform magnetic field and cold cylinder shell were also accounted. The obtained results propose that the average Nusselt number is an increasing function of the shape factor, Rayleigh number, and nanoparticles volume fraction. At the same time, it is reversely affected by the Hartmann number. The natural convective heat transfer and fluid flow were simulated by the lattice Boltzmann numerical method by Rahimi et al. [144]. The total entropy generation process and the free convective heat transfer were numerically analyzed using the Lattice Boltzmann method by Rahimi et al. [3]. It was concluded that the array of refrigerant bodies significantly influences the streamlines of isothermal lines. Lattice Boltzmann simulation was selected as the numerical tool to simulate the free convection in an H-shaped enclosure filled with nanofluid, which was performed by Rahimi et al. [145], where the arrangements of active internal bodies significantly affect the local entropy generation and Nusselt number. A model was developed to investigate the nanofluid free convection between a square cavity and a rectangular, an elliptical, or a circular cylinder by Roy [146]. The outcome of these investigations showed that the streamline's intensity increased with inner shapes of rectangular, circular, and elliptical. Also, the problem of free convection in a nanofluid-filled container with several solid structures was solved via an incompressible smoothed particle hydrodynamics (ISPH) method, Aly and Raizah [147]. The quantity of solid structures has a significant impact on the heat transfer rate and fluid flow inside a cavity, according to the findings.

The nanofluid flow and rate of heat transfer in a heat exchanger, using various thermal configurations of active interior bodies were studied by KhakRah et al. [148]. The obtained results convey that a better heat transfer performance is achieved because platelet nanoparticles result in a larger average Nusselt number. An investigation of fluid flow and heat transfer was conducted by Hussain and Rahomey [149]. The temperature gradient induced to flow between an inner hot cylinder and an outer cold square cavity filled with nanofluid superposed porous-nanofluid layers. As a consequence, the findings showed that the computed total surfaces-averaged Nusselt numbers of the cavity and the heat transfer rate from the outer cavity, in which the triangular cylinder is inserted, are the best. Natural convective heat transfer in an annulus between confocal elliptic cylinders filled with CNT-water nanofluid was numerically studied [150].

Free convection heat transfer and fluid flow with an interior circular cylinder of parallelogram cavity using Cu-water nanofluid were numerically investigated by Majdi et al. [151]. Flow patterns and heat transmission were found to be significantly affected by the inclination angle. A temperature gradient from the heated inner corrugated cylinder was used to simulate the natural convection of heat transmission by Abdulkadhim et al. [152]. Results suggested that moving upward the internal sinusoidal cylinder increases the strength of the fluid flow and the increase of the porous layer thickness reduces the average heat transfer. Free convective heat transfer between a hot triangular-shaped permeable cylinder and a cold square enclosure was examined under the influence of a magnetic field using the lattice Boltzmann method by Vijaybabu and Dhinakaran [153]. It saw that the permeability increment enhances the fluid momentum, whereas the magnetic field reduces the kinetic energy of the fluid.

In addition to the effect of fluid-solid thermal conductivity ratio, the magnetic field effect was numerically investigated on natural convection and generation of entropy in a nanofluid-filled cavity with a conducting wavy solid block by Tayebi and Chamkha [154]. The generation of entropy, patterns of fluid flow, and rate of heat transfer regarding Cu–$Al_2O_3$ -water hybrid nanofluids in an enclosure which has a complex shape and contains a hot-half partition were conducted by Alsabery et al. [155]. The obtained results demonstrated that using hybrid nanofluids enhances the Nusselt number compared with plain nanofluids. The study of Tayebi et al. [156] was conducted to extract the characteristics of free convective heat transfer, flow field, and entropy generation. To account for the IHG/A phenomena, the working fluid was Cu–Al2O3/H2O based hybrid nano liquid in an annulus, which was separated from the outside by two elliptic cylindrical walls. The study of Dogonchi et al. [157] investigated numerically heat transfer during natural convection in a square shaped enclosure embedded with a wavy circular heater filled with nanoparticles and subjected to an external magnetic field. As a consequence, it was shown that the heat transmission rate decreases as the nanoparticle volume fraction increases and the Rayleigh number decreases. However, the Hartmann number has a reverse effect on it.

A comprehensive, benchmark numerical simulation was carried out by Moria [158] to reveal the significance of porous layers in the improvement of natural convection of L-shape enclosure and, secondly, non-dimensionally optimize the location of blocks and porous layer to achieve the maximum heat transfer rate. The study of Patpatiya et al. [159] looked at how various working fluids (air, water, and (Ag-water) nanofluid) affected laminar and steady-state buoyancy-induced flow across an eccentrically heated square plate positioned along a horizontal center plane in a square hollow. Effects of the location of the cylinder and the Rayleigh number were investigated using an analogous temperature and flow field distribution. Magnetohydrodynamic free convection of nanofluid (Cu-water) in a wavy walled cavity enclosing a circular hot cylinder was studied via employing Galerkin-weighted residual formulation by Abdulkadhim et al. [160]. Results confirmed that the heat coefficient plays an important role in the Nusselt number at a low Rayleigh number, while its significance diminished when the Rayleigh number increased. Hosseinjani and Roohi [161] performed a survey on the problem of the free convective heat transfer about an elliptical hot cylinder located in a cold rhombus nanofluid-filled enclosure under a uniform magnetic field was numerically performed. The study of Aly et al. [162] dealt with free convection flow induced by rotating circular cylinders in a horizontal wavy enclosure fully filled with nanofluid. The results conveyed that temperature distributions of nanofluid flow can be raised by increasing the radius of the inner cylinder. In an annulus with two heat sources and an inner spinning circular cylinder, a numerical analysis of convective heat transport has been performed by Mirzaie and Lakzian [163]. The results showed that the presence of nanoparticles, rotating cylinder, and applying approximation via the Boussinesq method and water in the vicinity of the density inversion point enhances the rate of heat transfer.

### 3.2. Mixed convection with inner bodies (single-phase nanofluid)

Nanofluids consist naturally of water or thermal oil mixed with nanoparticles to obtain increased performance. The nanoparticles made of $Al_2O_3$, Fe, CuO, Cu, Al, Ag, $TiO_2$, and $SiO_2$ are the most used in scientific experiments. The term nanofluid is practiced for the first time. By the scientist Choi 1995, nanofluids are considered more conducive to heat than ordinary fluids. Early understanding of nanofluids associated that one of the most important properties of nanofluids is the thermal conductivity, hence their importance in experiments awaiting the transition to where the transition to heat.

Katleh et al. [165] examined several water-based nanoparticles, $Al_2O_3$, CuO, Ag, and $TiO_2$, filled in a square enclosure heated by a centered triangular cylinder and cooled from the right and top walls. The top wall moves to the right to induce effective mixing inside the enclosure. The governing equations were solved using the finite difference method. Oscillating problems arising from the pressure gradient term were overcome by transforming the equations to stream function-vorticity formulations. Brinkman model was adopted for the viscosity, while the thermal conductivity was predicted using the Patel model, which is proven experimentally and to consider the Brownian motion and the size of the nanoparticles. All types of the examined nanoparticles showed the positive role of nanofluid accompanying the friction-driven flow at the top of the enclosure. The Ag nanoparticles gave the highest Nusselt number, while $TiO_2$ nanoparticles were the worst. Smaller size nanoparticles augment Nusselt numbers better than big ones. They did not explain the main cause of this effect; however, it refers to the easy Brownian motion of the small nanoparticles.

Chatterjee et al. [166] studied the roles of passive nanofluid and the active moving boundaries on heat convective in a cavity. The moving boundaries were the cold top wall and an inner adiabatic cylinder that rotates counter-clockwise in a sense opposite the top wall's movement. The nanofluid was copper nanoparticles-water based, but the investigated volume fraction exceeded the dilute limit of Brinkman and Maxwell-Garnet models, i.e., they set it up to 20%, which predicts overestimated properties. They used the FLUENT package to solve the problem. The nanofluid augments the convective heat transfer and reduces the drag coefficient. The drag coefficient rises when the moving top wall and the rotating cylinder are in opposite directions. As can be expected, the rotation of the adiabatic centered cylinder played an adverse action on the Nusselt number; however, the authors did not include any comments about this remark. We are sure that the opposite motions of the top wall and the rotating cylinder produced competed for actions that resulted in a perturbed streamline.

Nevertheless, we wonder why the authors did not examine a consistent boundary motion. This wonder was removed with the study of Selimefendigil and Oztop [167], where they considered the cylinder's clockwise rotation, i.e., making an aiding motion of the moving boundary. As expected, the Nusselt number is significantly gained in this sense. They also included the externally applied magnetic field as a control tool for the heat transfer inside the cavity. Selimefendigil and Oztop [167] restricted their tests within the plausible limit of the volume fraction of nanoparticles of the Brinkman and Maxwell-Garnet model, i.e., 5% as a maximum volume fraction. The numerical appliance was the finite element method.

In their problem, Boulahia et al. [168] categorized two cases of lid-driven cavity including two hot triangular cylinders; namely, the bottom insulated walls move in either positive or negative x-directions. The vertical walls are kept cool while the top wall is also insulated and moves in a positive (right) direction. Amongst the three types of nanoparticles considered, Copper gave the highest Nusselt number while Titanium oxide was the worst. However, these two nanoparticles were not so deviate from the results of Alumina. The smaller the diameter of nanoparticles was demonstrated to be, the higher the Nusselt number. No significant variations were observed when the direction of the moving base was reversed. Alsabery et al. [169] formulated a problem regarding the entropy generation and convective heat transfer inside a cavity with wavy walls containing a rotating active cylinder. The cavity is filled with alumina-water nanofluid. The nanofluid is heated up isothermally from a segment centered in the base of the cavity while the wavy vertical walls are cooled isothermally. The Corcione model was adopted for the two main nanofluid properties: thermal conductivity and viscosity. The finite element method was used in the numerical analysis. Their results were focused on the entropy generation, where the rotation of the cylinder inferred the friction irreversibility. They found that the rotation of the cylinder is useless for high Rayleigh numbers; otherwise, unclear relation between the cylinder rotational speed and the other parameters. The waviness of the vertical walls induces many weak vortices. Thus, the friction irreversibility decreases compared with the thermal one. The volume fraction of the alumina raises the Nusselt number with an ignorable impact on the entropy generation.

Boulahia et al. [170] considered the obstruction effect of a cold cylinder centered inside a vented cavity. The base of the cavity heats up the nanofluid isothermally. The cold nanofluid enters the cavity from a port centered in the left vertical wall, while it leaves the cavity from a port located in the right vertical wall. Three locations of the outlet port were inspected. Unlike the bottom wall and ports, the walls are kept insulated. The volume fraction and Richardson number were studied for each outlet location with the aid of the finite volume method and the Corcione moles of nanofluid. It can be drawn from their results that when the outlet port is located in the upper or middle of the right wall, the role of nanoparticles scales down for weak and moderate natural convection. Locating the outlet port at the lower part of the right wall gives the best heat transfer. Hussein et al. [171] considered the CuO-water nanofluid in a trapezoidal cavity packed by a lower porous layer covered with regular fluid. A circular insulated cylinder is placed at the midline of the interface and rotates counterclockwise. The bottom of the cavity is wavy, and it heats up the domain isothermally. The side walls are adiabatic, while the top wall extracts the heat by cooling it with a constant temperature. The authors chose the primitive Maxwell-Garnett and Brinkman formulas to predict thermal conductivity and viscosity. They implemented COMSOL Multiphysics to solve the governing equations. Authors found common trends of increasing Nusselt number with the nanoparticles volume fraction, Darcy number, and cylinder rotational speed. While the effect of the rotating cylinder was much appreciated in the regular layer, the height of the porous layer weakens the convective heat transfer.

Thakur et al. [172] described the mixed convection of Cu-water nanofluid filled in a square cavity involving an isothermal hot circular cavity with all sides being cold. The top side moves in the positive x-direction. They inspected limited parameters: the volume fraction, Reynolds, and Grashof numbers. The appliance of their investigation was the Galerkin finite element method, and the nanofluid properties were interpreted using primitive models. The augmentation of the Nusselt number due to the Reynolds and Grashof numbers is much higher than that due to the addition of nanoparticles. It was reported that the effect of the nanoparticle to increase the heat transfer is decreased with rising the Reynolds and Grashof numbers. Jamal and Hussain [173] extended the problem of an inner rotating cylinder with a moving wall, which has been studied previously in several works, by considering a water-based hybrid nanofluid consisting of multi-walled carbon nanotube (MWCN)–iron oxide. To make their work different from others, they considered three different boundary conditions on the rotating cylinder: hot, cold, and adiabatic. Primitive Brinkman and Maxwell-Garnett relations with the mixing rules were adopted for nanofluid properties. The simulations were conducted using the Galerkin finite element. Their conclusions show no distinct fashion regarding the hybrid nanofluid compared with regular nanofluids. However, when the cylinder rotates incompatible with the top wall movement, higher Nusselt numbers were recorded.

The convective heat transport and produced entropy were studied by Ishak et al. [174] when they varied the size and placement of an active circular cylinder inside a trapezoidal cavity. The base is heated isothermally, the top wall is moving and cooled isothermally too, while the side walls are kept adiabatic. Alumina-water nanofluid is used to fill the cavity, and the Corcione model is used to predict its characteristics. Galerkin finite element technique was used to solve the conservation equations. Their results showed an increase in the convective heat transfer and entropy generation with the addition of the alumina nanoparticles. The size and location of the inner cylinder was the most dominant parameter affecting the heat transport and the entropy within the cavity. Ali et al. [175] conducted a numerical simulation on a triangular shed-shaped enclosure involving two adiabatic rotating cylinders saturated with alumina-water nanofluid. The source of the heat is a rectangular segment placed in three different locations at the base of the enclosure, and the shed walls are cooled isothermally. For the sake of control, a magnetic field is applied transversely. The adopted thermal conductivity model considers the high-speed motions of nanoparticles formulated by adding a dynamic term to the primitive Maxwell-Garnett model. The viscosity was modeled based on a polynomial relation (Pak and Cho). The governing equations were analyzed via the finite element method available in Comsol Multiphysics software. They reported a significant heat transfer increase by increasing the rotational speed and localizing the heat source at the corner of the enclosure. They reported a considerable percentage augmentation of the Nusselt number when raising the volume fraction to 5%.

A suppression role of the magnetic field upon the fluid circulation and heat transfer was observed. Shirani and Toghraie [176] solved the mixed convection and the entropy generation in an enclosure involving several objects. These are; a square porous zone confined in the center of the enclosure; four circular cylinders within the porous zone rotate in the same harmonic fashion. A copper-water nanofluid saturates the fluid and porous zones. The Maxwell-Garnett and Pak and Cho models were adopted to predict the thermal conductivity and the viscosity, respectively. The source of heat transfer was a hot base and cold top wall, moving to the right. The problem was solved using the finite volume method. Distinct results were found in their study: Nusselt number elevated with decreasing Darcy and Richardson numbers. They also highlighted that the porosity has no decisive impact on the Nusselt number, and an increasing permeability, the impact of the four-cylinder harmonic rotation is more apparent. The role of the nanoparticles was positive, where for a certain circumstance, setting the volume fraction at 3%, the Nusselt number rises to 30%.

## 4. Natural and mixed convection in enclosures with inner bodies (two-phase nanofluid model)

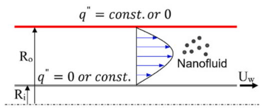
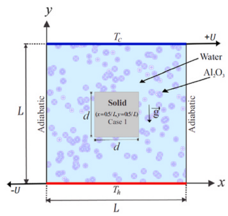
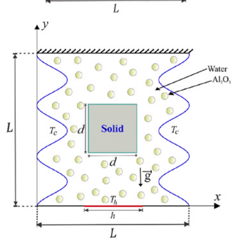
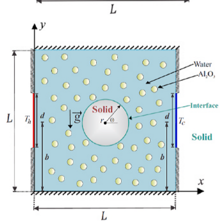
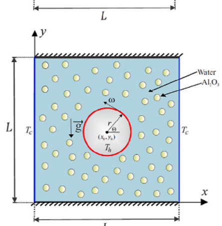
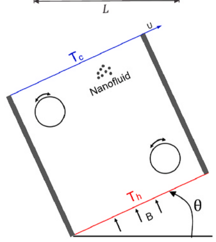
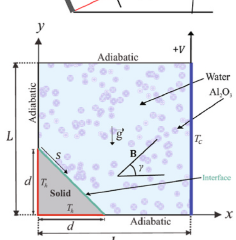
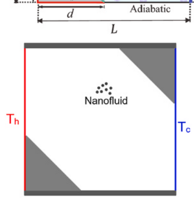
With the growing importance of nanofluids, more attention has been paid to discovering the mechanism of the suspension of nanoparticles within the base fluid. One of the most plausible explanations for such a mechanism is considering the nanoparticles and the base fluid as a separated phase. This model, which Buongiorno developed in 2006 [177], confirmed that the heat transfer could not only be enhanced by thermal conductivity but also due to the slip mechanism between the nanoparticles and the base fluid. Specifically, Buongiorno quoted two important mechanisms of the slip velocity: the Brownian motion, which originates from the random collisions between the molecules of the base fluid and the nanoparticles, and the thermophoresis, which results from the diffusion of the nanoparticles due to the temperature gradient. Four equations are to be solved to deal with such a model. Thus, it requires more computational effort. However, as we will see in the following reviews, researchers were aware of doing their studies within the laminar region as the Brownian motion diminishes in the existence of turbulent interferences.

Malvandi and Ganji [178] considered two thermal boundary conditions for the alumina-water nanofluid flow in a horizontal annulus with a moving core. The first set is a heat flux on the outer cylindrical wall and an insulating inner moving one. This arrangement of the boundary conditions is reversed in the second set. The study was restricted to the fully developed region. Thus, they transformed the governing equations into differential ones and used the Runge-Kutta-Fehlberg scheme. They concluded that the thermal boundary conditions strongly affect the nanoparticles' migration. When the moving core is heated, the momentum affects the concentrated nanoparticles there and then contributes to notable heat transfer enhancement. They also observed that lowering the ratio of Brownian to thermophoretic diffusivities leads to deforming the uniform distribution of the nanoparticles.

Alsabery et al. [179] adopted Buongiorno's model for inspecting the migration of alumina nanoparticles in water base fluid filling a square cavity, including a square conductive body. Corcione's empirical relations were implemented to predict the viscosity and thermal conductivity. Opposite movements and different temperatures were set on the horizontal walls. Their numerical results, which are collected using the Galerkin finite element method GFEM, showed that the nanoparticles concentrate around the inner square solid because it acts as a core of the weak roll. The nanoparticles are also concentrated because of thermophoresis diffusion close to the moving upper cold wall. This migration of nanoparticles conveys the energy between different zones of the cavity. They showed that in the case of low Reynolds number and high Richardson number, the alumina nanoparticles act adversely in transmitting the energy as the size of the block does. The block's location plays a notable role in controlling the friction effect of the moving walls. Using the same model and the numerical code, Hashim et al. [180] investigated the existence of the square conductive block inside a cavity that comprises a wavy vertical cold wall with a local heat source at the bottom wall. The hot natural convection rising from the local heat source and hitting the lower side of the inner block reduced the concentration of nanoparticles on this side. In contrast, contrariwise, the other sides exhibit higher concentrations. They assigned more uniformity of nanoparticles at a higher Rayleigh number, which significantly raises the Nusselt number.

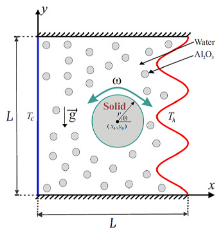
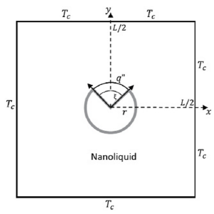
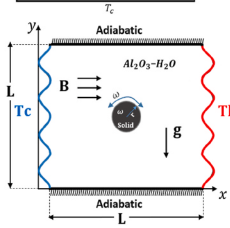
Alsabery et al. [181] continued with the model mentioned in Refs. [179,180], but the source and sink of heat are segmentally set on the vertical walls and a central circular cylinder. They noticed a reduction in the zones of high nanoparticle concentration as the size of the cylinder increases, while more uniformity is highlighted as its thermal conductivity increases. The nanoparticles enhance the heat transfer significantly at a low Rayleigh number. In another case, Alsabery et al. [182] developed the problem to involve the mixed convection arising from rotating the inner cylinder and maintaining it at a constant hot

**Table 5**
Natural and mixed convection in enclosures with inner bodies (Two-phase nanofluid model).

| Authors | Geometry | Solution method and used models | Results and remarks |
|---|---|---|---|
| Malvandi and Ganji [178] |   | Runge-Kutta-Fehlberg scheme formulated using FORTRAN. Corcione & Buongiorno's model | Nanoparticle movement is greatly influenced by thermal boundary conditions. Lowering the ratio of Brownian to thermophoretic diffusivities disturbs the uniform distribution of the nanoparticles. |
| Alsabery et al. [179] with permission from Elsevier |   | Galerkin FEM. Buongiorno's model & Corcione model | The nanoparticles concentrate around the inner square solid and close to the moving upper cold wall. This migration of nanoparticles conveys the energy between different zones of the cavity. |
| Hshim et al. [180] with permission from Elsevier |   | Galerkin FEM. Buongiorno's model & Corcione model | Except for the lower side of the inner block, the concentration of nanoparticles concentrates along the other sides of it. More uniformity of nanoparticles is assigned at a higher Rayleigh number, which significantly raises the Nusselt number. |
| Alsabery et al. [181] with permission from Elsevier |   | Galerkin FEM. Buongiorno's model & Corcione model | The zones of high nanoparticles concentration dwindle as the size of the cylinder increases, while more uniformity is highlighted as its thermal conductivity increases. The nanoparticles enhance the heat transfer significantly at a low Rayleigh number. |
| Alsabery et al. [182] with permission from Springer Nature |   | Galerkin FEM. Buongiorno's model | The nanoparticle's distribution is uniform for the motionless inner cylinder while they concentrate in the left half of the rotating cylinder. For higher speed, a localized high concentration of nanoparticles and a strong source of entropy generation zone are seen on the left side of the cylinder. |
| Re-plotted from Barnoon et al. [183] |   | Finite volume method. Buongiorno's model & Maxwell and Brinkman's model | The isothermal cold boundary of the rotating cylinders is better in heat exchange than the adiabatic one. An important role of the existence and the speed of rotating cylinders in enhancing the heat transfer. The magnetic field suppresses the rate of entropy generation. |
| Alsabery et al. [184] |   | Galerkin FEM. Buongiorno's model & Corcione model | Higher concentrations of the nanoparticles were collected close to two zones; these are the left upper corners of the cavity and the triangular solid. They concluded that the magnetic field enhances the Brownian and thermophoresis diffusions. However, no clear conclusion was made about the magnetic field impact on the average Nusselt number. |
| Re-plotted from Zadeh et al. [185] |   | Galerkin FEM. Buongiorno's model & linear viscosity and linear thermal conductivity | The concentration of the nanoparticles elevated with increasing the size of the two solids, which boosted the Nusselt number. The cold vertical wall shows a higher concentration zone. |

(*continued on next page*)

**Table 5** (*continued*)

| Authors | Geometry | Solution method and used models | Results and remarks |
|---|---|---|---|
| Alsabery et al. [187] |  | Galerkin FEM. Buongiorno's model & Corcione model | The wavy hot wall leads to trapping nanoparticles along the undulations, and this restricts the transport of energy; thus, the Nusselt number decreases with the number of undulations. There was no clear relation between the size of the inner cylinder and the heat transfer. However, with respect to the wavy wall, the counterclockwise rotation offered the best heat transfer than the clockwise rotation. |
| Alhashash [188] |  | Galerkin FEM. Buongiorno's model & Corcione model | The nanoparticles concentrate along the heated portion. The heat transfer rate drops with the size of the cylinder, and the worst heat transfer rate was recorded when the heated portion occupies one-fourth of the cylinder surface |
| Al-Kouz et al. [189] |  | Galerkin FEM. Buongiorno's model & Corcione model | Nusselt number decays with the waviness of the vertical walls, which is mainly due to the high concentration of nanoparticles. The counterclockwise rotation of the cylinder showed better heat transfer than the opposite rotation. The heat transfer is the main cause of raising the thermodynamic irreversibility (see Table 5). |

temperature while the vertical walls are maintained at a cold temperature. They took into their computation the generated entropy due to heat transfer and fluid friction irreversibility. Their results showed that the nanoparticle distribution is uniform for the motionless inner cylinder. At the same time, they concentrate on the left half of the cylinder surface when the cylinder rotates at moderate speed. For higher speed, a localized high concentration zone is seen on the left side of the cylinder. This collection of the nanoparticles resulted from the opposite actions of the natural convection and the friction force of the counterclockwise rotation of the cylinder. The entropy generation was seen in the higher concentration nanoparticles' zones, which implies the friction irreversibility.

Barnoon et al. [183] considered the mixed convection and entropy generation in a square cavity involving two rotating cylinders localized at two opposite corners of the cavity. The bottom wall is isothermally heated, while the top wall is considered cold and moving to the right. A magnetic field is applied from below while several inclinations of the whole cavity are considered. The finite volume method was implemented in the solution, and primitive Brinkman and Maxwell relations to predict the viscosity and thermal conductivity were adopted, respectively. Buongiorno's model was adopted for thermal and random diffusions. Two conditions on the rotating cylinders have been tested. The isothermal cold boundary offered better heat exchange than the adiabatic one. They indicated an important role of the existence and the speed of rotating cylinders in enhancing the heat transfer. The magnetic field suppresses the rate of entropy generation.

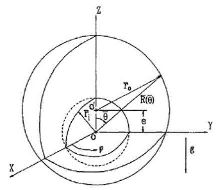
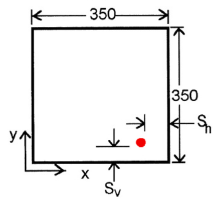
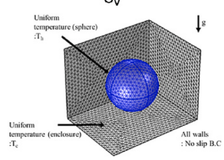
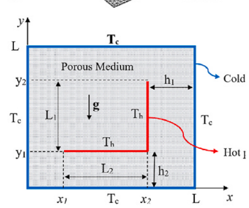
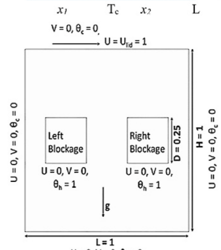
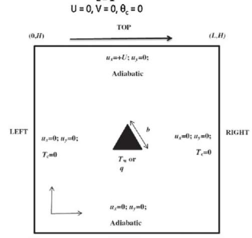
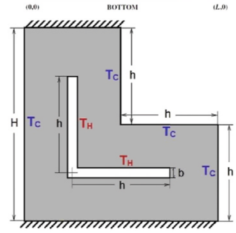
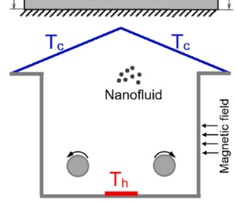
Alsabery et al. [184] imposed the conjugate heat transfer by injecting the energy through a triangular solid fixed at the left lower corner of a square cavity with an upward-moving right vertical wall, which is kept at a cold temperature. The whole domain is subjected to a magnetic field. The slip assumption and nanofluid relation are the same in his works [179–183]. Due to the counter-rotating rolls inside the cavity, a higher concentration of the nanoparticles was collected close to two zones; these are the left upper corners of the cavity and the triangular solid. An increase in the migration of the nanoparticles was observed with the augmentation of the magnetic field. Thus, they concluded that the magnetic field enhances the Brownian and thermophoresis diffusions. However, no clear conclusion was made about the magnetic field impact on the average Nusselt number.

It has been shown that natural convection may occur in a square cavity when two conductor triangular solids are placed at the bottom and top of it, respectively, by Zadeh et al. [185]. Both of the vertical cavity walls were differentially heated. The properties of the viscosity and the thermal conductivity of nanofluid were taken as linear variations according to the benchmark study of Buongiorno [186]. The calculations were achieved by Galerkin FEM. The concentration of the nanoparticles was elevated by increasing the two solids' size, which boosted the Nusselt number. The cold vertical wall shows a higher concentration zone.

Alsabery et al. [187] modified the square cavity involving a rotating conductive cylinder by undulating the hot vertical wall while keeping the other vertical wall cold and flat. Following their numerical code, they implemented the Galerkin FEM and Buongiorno's model with Corcione correlations for the viscosity and the thermal conductivity of the alumina-water nanofluid. They revealed that the undulation nature of the hot wavy wall leads to trapping nanoparticles along the undulations, which restricts energy transport. Therefore, the Nusselt number was a decreasing function of the number of undulations. Indeed, the concentration of nanoparticles follows a random behavior with the size of the rotating cylinder. As a result, there was no clear relation with the heat transfer. However, concerning the wavy wall, the counterclockwise rotation offered the best heat transfer than the clockwise rotation.

Alhashash [188] is considered a porous square enclosure saturated with alumina-water nanofluid and involving an adiabatic circular cylinder. A segmental heater is assumed to occupy a portion of the surface of the cylinder. All the external walls of the enclosure were cooled at a constant temperature. The Galerkin FEM and the assembly models of Buongiorno's and Corcione were adopted. The indicated results showed that the nanoparticles concentrated highly along the heated portion. The heat transfer rate drops with increasing the size of the cylinder, and the worst heat transfer rate was recorded when the heated portion occupies one-fourth of the cylinder surface. Al-Kouz et al. [189] implemented the set of Buongiorno's and Corcione to simulate the mixed convection inside a wavy vertical walled cavity with an adiabatic rotating cylinder. The cavity is differentially heated and subjected to a transverse magnetic

**Table 6**
Correlations of the average Nusselt number and some physical properties.

| Authors | Geometry | Conditions | Relations |
|---|---|---|---|
| Chiu and Chen [20] |  | For centered cylinders:<br>For eccentric (upper) cylinders:<br>For eccentric (lower) cylinders: | $Nu = 0.210Ra^{0.240}$<br>$Nu = 0.267Ra^{0.209}$<br>$Nu = 0291Ra^{0.211}$ |
| Ekundayo et al. [25] |  | For Ra = $10^5$, X = x/350 = 0.5: where * stands for centered heater<br>For Ra = $10^5$, X = x/350 = 0.9: | $\frac{Nu}{Nu^*} = 0.34 + 17Y - 57Y^2 + 46.4Y^3 - 35Y^4$<br>$\frac{Nu}{Nu^*} = 1 - 8Y + 36Y^2 - 54Y^3 + 26Y^4$ |
| Lee et al. [96] |  | κ, $κ_1$, $κ_2$ and n are complicated functions to the dimensions of the enclosure L and W. | $Nu = \kappa_1 + (\kappa_2 - \kappa_1)\frac{Ra^n}{\kappa + Ra^n}$ |
| Alomar et al. [99] |  | Ra* (100–1000), Fs/Pr*($10^{-4}$ –$10^{-2}$), H (0.1–100) and Kr(0.1–100) | $Nu = 0.217928(Ra^*)^{0.370329} \times \left(\frac{Fs}{Pr^*}\right)^{-0.49392} \times (H)^{0.00347} \times (Kr)^{0.100337}$ |
| Morshed et al. [121] |  | Case I: The blockages are horizontally aligned<br>Case II: The left blockage at the lower left corner while the right one at the upper right corner<br>Case III: The left blockage at the upper left corner while the right one at the lower right corner | $Nu = 0.3738 \times Re^{1.1161}Ri^{0.0021}$<br>$Nu = 2.5741 \times Re^{0.0412}Ri^{0.0363}$<br>$Nu = 2.5549 \times Re^{0.0460}Ri^{0.0448}$ |
| Gangawane and Manikandan [122] |  | For constant wall temperature<br>For constant heat flux | $Nu = e\left[\begin{array}{c}8.036 \times 10^{-6}Gr+ \\ 7.848 \times 10^{-3}\,Pr - 6.932 \times 10^{-4}Re + 1.073\end{array}\right]$<br>$Nu = e\left[\begin{array}{c}9.740 \times 10^{-6}Gr+ \\ 6.721 \times 10^{-3}\,Pr - 3.925 \times 10^{-4}Re + 1.187\end{array}\right]$ |
| Rahimi et al. [3] |  | For φ = 2%, the viscosity is:<br>For φ = 2%, the thermal conductivity is: | $\mu = 4.8547 - 0.0691T + 0.0003T^2$<br>$k = 0.4421 - 5 \times 10^{-5}T + 10^{-5}T^2$ |
| Ali et al. [175] |  | $1 \le Re \le 100$<br>$0 \le Ha \le 50$<br>$1 \le Re \le 100$<br>$0\% \le \varphi \le 5\%$ | $Nu = 5.79852827 + 0.18435936Re - 0.09842105Ha$<br>$Nu = 2.4925926 + 0.1888188Re + 0.6416667\varphi$ |

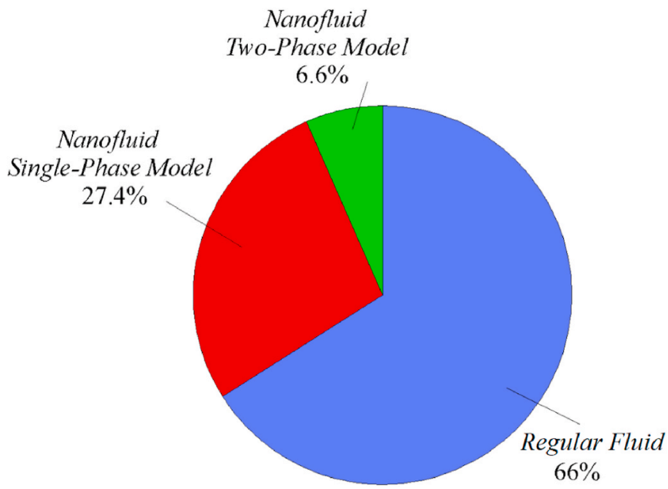



**Fig. 4.** Percentages of all studies of cavities involving inner solid.

field. Their tool of computation was the Galerkin FEM. The reported results also implicated the drop of Nusselt number with the waviness of the vertical walls, which is mainly due to the collection of a high concentration of nanoparticles. The cylinder's aiding rotation (counterclockwise) showed better heat transfer than the opposite rotation. The heat transfer was the main cause of raising the thermodynamic irreversibility.

## 5. Correlations

The current review has revealed little attention in the correlations of heat transfer. The reason may be attributed to that the authors focused on the correlations of the thermophysical properties. As such, few papers considered the correlations the average Nusselt number with various parameters. Table 6 summarizes most of the available correlations of the Nusselt number. The effort of Rahimi et al. [144], which focused of correlating the viscosity and the thermal conductivity with the temperature is also presented in the table.

## 6. Summary and conclusions

This review focuses on the natural and mixed convection in cavities, including solid inner bodies. The review included two types of working fluids that fill the cavities, namely regular and nanofluids. Because they give distinct explanations for the nanoparticle's role, we distinguished between the single-phase and two-phase models. Accordingly, the review is divided into three main sections, and the works are reviewed chronologically. The review has triggered the following conclusions.

1. The present review has revealed a large number of studies regarding the cavity, including an inner solid, rotating or fixed, adiabatic or thermally conductive, filled with regular and nanofluids. This emphasizes the importance of such geometry in simulating the heat exchanges, cooling ingots, or controlling the heat transfer. However, amongst these huge studies, we identified that 34% of these have considered nanofluids. Regarding the nanofluids studies only, about 19% of these have adopted the two-phase Buongiorno's model, even though it gives an actual description of nanoparticles' diffusion. Figs. 4 and 5 portray statistical details of the percentages ratio of various studies. We think that the extra numerical efforts are the main reason behind the aversion of researchers to this model.
2. In general, studies investigating a vented cavity filled with nanofluids and involving an inner block are very scarce. This refers to the toxicity nature of nanofluid, which necessitates the use of a closed system.
3. The rotation of the inner cylinders becomes useless when the natural convection is dominant, i.e., at higher Rayleigh and Grashof numbers. Moreover, the rotation gives rise the frictional irreversibility.
4. Nanoparticles concentrate along the surface of the inner blocks. Thus the larger sizes of inner blocks decline the heat transfer in most reviewed studies.
5. Most of the studies regarding the two-phase Buongiorno's model have used alumina nanoparticles (Al2O3) based water. This is because Buongiorno [169] established most of his pioneer models based on alumina-water nanofluid.
6. Based on Buongiorno's model, we do not recommend the wavy-walled cavities because the nanoparticles concentrate on the horizontal segments of the undulations and consequently lower heat transfer exchange.

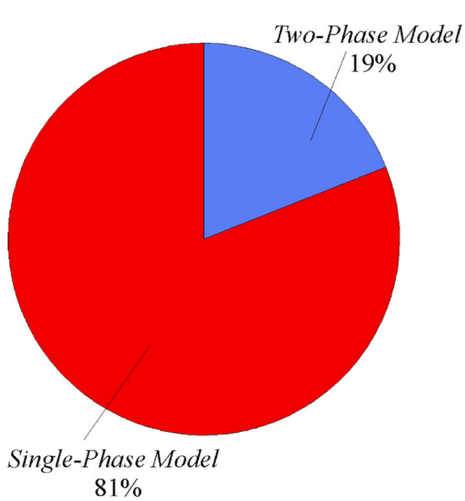



**Fig. 5.** Percentages of nanofluid studies only.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

No data was used for the research described in the article.

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## References

[1] Rahimi A, Kasaeipoor A, Malekshah EH. Lattice Boltzmann simulation of natural convection and entropy generation in cavities filled with nanofluid in existence of internal rigid bodies-Experimental thermo-physical properties. J Mol Liq 2017; 242:580–93.
[2] Rahimi A, Kasaeipoor A, Malekshah EH, Amiri A. Natural convection analysis employing entropy generation and heatline visualization in a hollow L-shaped cavity filled with nanofluid using lattice Boltzmann method- experimental thermo-physical properties. Phys E Low-Dimensional Syst Nanostructures 2018; 97:82–97.

[3] Rahimi A, Kasaeipoor A, Malekshah EH, Palizian M, Kolsi L. Lattice Boltzmann numerical method for natural convection and entropy generation in cavity with refrigerant rigid body filled with DWCNTs-water nanofluid-experimental thermo-physical properties. Therm Sci Eng Prog 2018;5:372–87.
[4] Sheremet MA, Oztop HF, Pop I, Abu-Hamdeh N. Analysis of entropy generation in natural convection of nanofluid inside a square cavity having hot solid block: tiwari and das' model. Entropy 2016;18:9.
[5] Buongiorno J. Convective transport in nanofluids. J Heat Tran 2006;128:240–50.
[6] Mehta B, Subhedar D, Panchal H, Said Z. Synthesis, stability, thermophysical properties and heat transfer applications of nanofluid – a review. J Mol Liq 2022;364:120034.
[7] Saha G, Al-Waaly AAY, Paul MC, Saha SC. Heat transfer in cavities: configurative systematic review. Energies 2023;16:2338.
[8] Lei M, Kleinstreuer C. Natural-convection heat transfer in a nonuniform finite annulus with concentric heat source. Int J Heat Fluid Flow 1994;15:456–61.
[9] Burns AS, Stewart WE. Convection in heat generating porous media in a concentric annulus with a permeable outer boundary. Int Commun Heat Mass Tran 1992;19:127–36.
[10] Stewart WE, Burns AS. Convection in a concentric annulus with heat generating porous media and a permeable inner boundary. Int Commun Heat Mass Tran 1992;19:859–68.
[11] Wang QW, Tao WQ. Natural convection in square enclosure with an internal isolated horizontal plate. Int J Fluid Mech Res 1998;25:546–55.
[12] Ghaddar NK. Natural convection heat transfer between a uniformly heated cylindrical element and its rectangular enclosure. Int J Heat Mass Tran 1992;35:2327–34.
[13] Sawada T, Kikura H, Saito A, Tanahashi T. Natural convection of a magnetic fluid in concentric horizontal annuli under nonuniform magnetic fields. Exp Therm Fluid Sci 1993;7:212–20.
[14] Ho CJ, Cheng YT, Wang CC. Natural convection between two horizontal cylinders inside a circular enclosure subjected to external convection. Int J Heat Fluid Flow 1994;15:299–306.
[15] Fu WS, Cheng CS, Shieh WJ. Enhancement of natural convection heat transfer of an enclosure by a rotating circular cylinder. Int J Heat Mass Tran 1994;37:1885–97.
[16] Lei M, Kleinstreuer C. Natural-convection heat transfer in a nonuniform finite annulus with concentric heat source. Int J Heat Fluid Flow 1994;15:456–61.
[17] Moukalled F, Acharya S. Natural convection in the annulus between concentric horizontal circular and square cylinders. J Thermophys Heat Tran 1996;10:524–31.
[18] Liu Y, Phan-Thien N, Kemp R. Coupled conduction-convection problem for a cylinder in an enclosure. Comput Mech 1996;18:429–43.
[19] Lacroix M, Joyeux A. Coupling of wall conduction with natural convection from heated cylinders in a rectangular enclosure. Int Commun Heat Mass Tran 1996;23:143–51.
[20] Chiu CP, Chen WR. Transient natural convection between concentric and vertically eccentric spheres with mixed boundary conditions. Heat Mass Transf Und Stoffuebertragung 1996;31:137–43.
[21] Oh JY, Ha MY, Kim KC. Numerical study of heat transfer and flow of natural convection in an enclosure with a heat-generating conducting body. Numer Heat Tran 1997;31:289–303.
[22] Sun YS, Emery AF. Effects of wall conduction, internal heat sources and an internal baffle on natural convection heat transfer in a rectangular enclosure. Int J Heat Mass Tran 1997;40:915–29.
[23] Char MI, Lee GC. Maximum density effects on natural convection of micropolar fluids between horizontal eccentric cylinders. Int J Eng Sci 1998;36:157–69.
[24] Sasaguchi K, Kuwabara K, Kusano K, Kitagawa H. Transient cooling of water around a cylinder in a rectangular cavity - a numerical analysis of the effect of the position of the cylinder. Int J Heat Mass Tran 1998;41:3149–56.
[25] Ekundayo CO, Probert SD, Newborough M. Heat transfers from a horizontal cylinder in a rectangular enclosure. Appl Energy 1998;61:57–78.
[26] Ha MY, Jung MJ, Kim YS. Numerical study on transient heat transfer and fluid flow of natural convection in an enclosure with a heat-generating conducting body. Numer Heat Tran 1999;35:415–33.
[27] Cesini G, Paroncini M, Cortella G, Manzan M. Natural convection from a horizontal cylinder in a rectangular cavity. Int J Heat Mass Tran 1999;42:1801–11.
[28] Ha MY, Jung MJ. A numerical study on three-dimensional conjugate heat transfer of natural convection and conduction in a differentially heated cubic enclosure with a heat-generating cubic conducting body. Int J Heat Mass Tran 2000;43:4229–48.
[29] Shu C, Xue H, Zhu YD. Numerical study of natural convection in an eccentric annulus between a square outer cylinder and a circular inner cylinder using DQ method. Int J Heat Mass Tran 2001;44:3321–33.
[30] Ha MY, Kim IK, Yoon HS, Yoon KS, Lee JR, Balachandar S, et al. Two-dimensional and unsteady natural convection in a horizontal enclosure with a square body. Numer Heat Tran 2002;41:183–210.
[31] Shu C, Zhu YD. Efficient computation of natural convection in a concentric annulus between an outer square cylinder and an inner circular cylinder. Int J Numer Methods Fluid 2002;38:429–45.
[32] Tasnim SH, Mahmud S, Das PK. Effect of aspect ratio and eccentricity on heat transfer from a cylinder in a cavity. Int J Numer Methods Heat Fluid Flow 2002;12:855–69.
[33] Roychowdhury DG, Das SK, Sundararajan T. Numerical simulation of natural convective heat transfer and fluid flow around a heated cylinder inside an enclosure. Heat Mass Transf Und Stoffuebertragung 2002;38:565–76.
[34] Peng Y, Chew YT, Shu C. Numerical simulation of natural convection in a concentric annulus between a square outer cylinder and a circular inner cylinder using the Taylor-series-expansion and least-squares-based lattice Boltzmann method. Phys Rev E 2003;67:6.
[35] Lee JR, Ha MY. A numerical study of natural convection in a horizontal enclosure with a conducting body. Int J Heat Mass Tran 2005;48:3308–18.
[36] Ding H, Shu C, Yeo KS, Lu ZL. Simulation of natural convection in eccentric annuli between a square outer cylinder and a circular inner cylinder using local MQ-DQ method. Numer Heat Tran 2005;47:291–313.
[37] Lee JR, Ha MY. Numerical simulation of natural convection in a horizontal enclosure with a heat-generating conducting body. Int J Heat Mass Tran 2006;49:2684–702.
[38] Kumar De A, Dalal A. A numerical study of natural convection around a square, horizontal, heated cylinder placed in an enclosure. Int J Heat Mass Tran 2006;49:4608–23.
[39] Bouafia M, Daube O. Natural convection for large temperature gradients around a square solid body within a rectangular cavity. Int J Heat Mass Tran 2007;50:3599–615.
[40] Liu D, Zhao FY, Tang GF. Conjugate heat transfer in an enclosure with a centered conducting body imposed sinusoidal temperature profiles on one side. Numer Heat Tran 2008;53:204–23.
[41] Jami M, Mezrhab A, Bouzidi M, Lallemand P. Lattice Boltzmann method applied to the laminar natural convection in an enclosure with a heat-generating cylinder conducting body. Int J Therm Sci 2007;46:38–47.
[42] Jami M, Mezrhab A, Naji H. Numerical study of natural convection in a square cavity containing a cylinder using the lattice Boltzmann method. Eng Comput 2008;25:480–9.
[43] Angeli D, Levoni P, Barozzi GS. Numerical predictions for stable buoyant regimes within a square cavity containing a heated horizontal cylinder. Int J Heat Mass Tran 2008;51:553–65.
[44] Saha S, Saha G, Islam M. Natural convection in square enclosure with adiabatic cylinder at center and discrete bottom heating. Daffodil Int Univ J Sci Technol 2008;3:29–36.
[45] Lu J, Shi B, Guo Z, Chai Z. Numerical study on natural convection in a square enclosure containing a rectangular heated cylinder. Front Energy Power Eng China 2009;3:373–80.
[46] Yoon HS, Ha MY, Kim BS, Yu DH. Effect of the position of a circular cylinder in a square enclosure on natural convection at Rayleigh number of 107. Phys Fluids 2009;21:0.
[47] Hussain SH, Hussein AK. Numerical investigation of natural convection phenomena in a uniformly heated circular cylinder immersed in square enclosure filled with air at different vertical locations. Int Commun Heat Mass Tran 2010;37:1115–26.
[48] Butler C, Newport D, Geron M. Natural convection experiments on a heated horizontal cylinder in a differentially heated square cavity. Exp Therm Fluid Sci 2013;44:199–208.
[49] Bararnia H, Soleimani S, Ganji DD. Lattice Boltzmann simulation of natural convection around a horizontal elliptic cylinder inside a square enclosure. Int Commun Heat Mass Tran 2011;38:1436–42.
[50] Yoon HS, Jung JH, Park YG. Natural convection in a square enclosure with two horizontal cylinders. Numer Heat Tran 2012;62:701–21.
[51] Park YG, Yoon HS, Ha MY. Natural convection in square enclosure with hot and cold cylinders at different vertical locations. Int J Heat Mass Tran 2012;55:7911–25.
[52] Liao CC, Lin CA. Influences of a confined elliptic cylinder at different aspect ratios and inclinations on the laminar natural and mixed convection flows. Int J Heat Mass Tran 2012;55:6638–50.
[53] Kalyana Raman S, Arul Prakash K, Vengadesan S. Natural convection from a heated elliptic cylinder with a different axis ratio in a square enclosure. Numer Heat Tran 2012;62:639–58.
[54] Kang DH, Ha MY, Yoon HS, Choi C. Bifurcation to unsteady natural convection in square enclosure with a circular cylinder at Rayleigh number of 107. Int J Heat Mass Tran 2013;64:926–44.
[55] Butler C, Newport D, Geron M. Natural convection experiments on a heated horizontal cylinder in a differentially heated square cavity. Exp Therm Fluid Sci 2013;44:199–208.
[56] Lee HJ, Doo JH, Ha MY, Yoon HS. Effects of thermal boundary conditions on natural convection in a square enclosure with an inner circular cylinder locally heated from the bottom wall. Int J Heat Mass Tran 2013;65:435–50.
[57] Park YG, Ha MY, Yoon HS. Study on natural convection in a cold square enclosure with a pair of hot horizontal cylinders positioned at different vertical locations. Int J Heat Mass Tran 2013;65:696–712.
[58] Park HK, Ha MY, Yoon HS, Park YG, Son C. A numerical study on natural convection in an inclined square enclosure with a circular cylinder. Int J Heat Mass Tran 2013;66:295–314.
[59] Liao CC, Lin CA. Transitions of natural convection flows in a square enclosure with a heated circular cylinder. Appl Therm Eng 2014;72:41–7.
[60] Park YG, Ha MY, Choi C, Park J. Natural convection in a square enclosure with two inner circular cylinders positioned at different vertical locations. Int J Heat Mass Tran 2014;77:501–18.
[61] Yoon HS, Park YG, Jung JH. Natural convection in a square enclosure with differentially heated two horizontal cylinders. Numer Heat Tran 2014;65:302–26.
[62] Choi C, Jeong S, Ha MY, Yoon HS. Effect of a circular cylinder's location on natural convection in a rhombus enclosure. Int J Heat Mass Tran 2014;77:60–73.

[63] Lam PAK, Arul Prakash K. A numerical study on natural convection and entropy generation in a porous enclosure with heat sources. Int J Heat Mass Tran 2014;69:390–407.
[64] Dash SM, Lee TS, Huang H. Natural convection from an eccentric square cylinder using a novel flexible forcing IB-LBM method. Numer Heat Tran 2014;65:531–55.
[65] Dash SM, Lee TS. Natural convection in a square enclosure with a square heat source at different Horizontal and diagonal eccentricities. Numer Heat Tran 2015;68:686–710.
[66] Liao CC, Lin CA. Influence of Prandtl number on the instability of natural convection flows within a square enclosure containing an embedded heated cylinder at moderate Rayleigh number. Phys Fluids 2015;27.
[67] Choi C, Cho HW, Ha MY, Yoon HS. Effect of circular cylinder location on three-dimensional natural convection in a cubical enclosure. J Mech Sci Technol 2015;29:1307–18.
[68] Park YG, Ha MY, Park J. Natural convection in a square enclosure with four circular cylinders positioned at different rectangular locations. Int J Heat Mass Tran 2015;81:490–511.
[69] Choi C, Ha MY, Park YG. Characteristics of thermal convection in a rectangular channel with an inner cold circular cylinder. Int J Heat Mass Tran 2015;84:955–73.
[70] Mun GS, Doo JH, Ha MY. Thermo-dynamic irreversibility induced by natural convection in square enclosure with inner cylinder. Part-I: effect of tilted angle of enclosure. Int J Heat Mass Tran 2016;97:1102–19.
[71] Doo JH, Mun GS, Ha MY, Seong SY. Thermo-dynamic irreversibility induced by natural convection in square enclosure with inner cylinder. Part-II: effect of vertical position of inner cylinder. Int J Heat Mass Tran 2016;97:1120–39.
[72] Lee SH, Seo YM, Yoon HS, Ha MY. Three-dimensional natural convection around an inner circular cylinder located in a cubic enclosure with sinusoidal thermal boundary condition. Int J Heat Mass Tran 2016;101:807–23.
[73] Seo YM, Doo JH, Ha MY. Three-dimensional flow instability of natural convection induced by variation in radius of inner circular cylinder inside cubic enclosure. Int J Heat Mass Tran 2016;95:566–78.
[74] Zhang P, Zhang X, Deng J, Song L. A numerical study of natural convection in an inclined square enclosure with an elliptic cylinder using variational multiscale element free Galerkin method. Int J Heat Mass Tran 2016;99:721–37.
[75] Zhang DD, Zhang JH, Liu D, Zhao FY, Wang HQ, Li XH. Inverse conjugate heat conduction and natural convection inside an enclosure with multiple unknown wall heating fluxes. Int J Heat Mass Tran 2016;96:312–29.
[76] Hu JT, Ren XH, Liu D, Zhao FY, Wang HQ. Conjugate natural convection inside a vertical enclosure with solid obstacles of unique volume and multiple morphologies. Int J Heat Mass Tran 2016;95:1096–114.
[77] Lage JL, Junqueira SLM, De Lai FC, Franco AT. Aspect ratio effect on the prediction of boundary layer interference in steady natural convection inside heterogeneous enclosures. Int J Heat Mass Tran 2016;92:940–7.
[78] Souayeh B, Ben-Cheikh N, Ben-Beya B. Numerical simulation of three-dimensional natural convection in a cubic enclosure induced by an isothermally-heated circular cylinder at different inclinations. Int J Therm Sci 2016;110:325–39.
[79] Hussein AK, Ashorynejad HR, Sivasankaran S, Kolsi L, Shikholeslami M, Adegun IK. Modeling of MHD natural convection in a square enclosure having an adiabatic square shaped body using Lattice Boltzmann Method. Alex Eng J 2016;55:203–14.
[80] Seo YM, Park YG, Kim M, Yoon HS, Ha MY. Two-dimensional flow instability induced by natural convection in a square enclosure with four inner cylinders. Part I: effect of horizontal position of inner cylinders. Int J Heat Mass Tran 2017;113:1306–18.
[81] Seo YM, Mun GS, Park YG, Ha MY. Two-dimensional flow instability induced by natural convection in a square enclosure with four inner cylinders. Part II: effect of various positions of inner cylinders. Int J Heat Mass Tran 2017;113:1319–31.
[82] Das D, Roy M, Basak T. Studies on natural convection within enclosures of various (non-square) shapes – a review. Int J Heat Mass Tran 2017;106:356–406.
[83] Jelita M, Mudia H, Afriani S. Analysis of fluid dynamics and heat transfer inside an elliptical cylinder in a square enclosure. Adv Theor Appl Math 2017;10:11–20.
[84] Xu HT, Wang TT, Qu ZG, Chen J, Li BB. Lattice Boltzmann simulation of the double diffusive natural convection and oscillation characteristics in an enclosure filled with porous medium. Int Commun Heat Mass Tran 2017;81:104–15.
[85] Kefayati GHR, Tang H. Double-diffusive natural convection and entropy generation of Carreau fluid in a heated enclosure with an inner circular cold cylinder (Part I: heat and mass transfer). Int J Heat Mass Tran 2018;120:731–50.
[86] Kefayati GHR, Tang H. Lattice Boltzmann simulation of viscoplastic fluids on natural convection in an inclined enclosure with inner cold circular/elliptical cylinders (Part I: one cylinder). Int J Heat Mass Tran 2018;123:1138–62.
[87] Kefayati GR, Tang H. Lattice Boltzmann simulation of viscoplastic fluids on natural convection in inclined enclosure with inner cold circular/elliptical cylinders (Part II: two cylinders). Int J Heat Mass Tran 2018;123:1163–81.
[88] Kefayati GHR, Tang H. Lattice Boltzmann simulation of viscoplastic fluids on natural convection in inclined enclosure with inner cold circular/elliptical cylinders (Part III: four cylinders). Int J Heat Mass Tran 2018;123:1182–203.
[89] Kefayati GHR, Tang H. MHD thermosolutal natural convection and entropy generation of Carreau fluid in a heated enclosure with two inner circular cold cylinders, using LBM. Int J Heat Mass Tran 2018;126:508–30.
[90] Aberuee MJ, Ziaei-Rad M, Ahmadikia H. The effects of an internal isolated plate on natural convective heat transfer inside an industrial oven. Appl Therm Eng 2018;144:769–78.
[91] Cho HW, Park YG, Ha MY. The natural convection in a square enclosure with two hot inner cylinders, Part I: the effect of one elliptical cylinder with various aspect ratios in a vertical array. Int J Heat Mass Tran 2018;125:815–27.
[92] Mun GS, Seo YM, Park YG, Ha MY. Natural convection in a cold enclosure with four hot inner cylinders in a diamond array (Part-II: effect of unequal horizontal and vertical distances of inner cylinders). Int J Heat Mass Tran 2018;120:1365–73.
[93] Seo YM, Ha MY, Park YG. The effect of four elliptical cylinders with different aspect ratios on the natural convection inside a square enclosure. Int J Heat Mass Tran 2018;122:491–503.
[94] Wang L, Zhao Y, Yang X, Shi B, Chai Z. A lattice Boltzmann analysis of the conjugate natural convection in a square enclosure with a circular cylinder. Appl Math Model 2019;71:31–44.
[95] Seo YM, Ha MY, Park YG. A numerical study on the three-dimensional natural convection with a cylinder in a long rectangular enclosure. Part I: size effect of a circular cylinder or an elliptical cylinder. Int J Heat Mass Tran 2019;134:420–36.
[96] Lee D, Jang H, Lee BJ, Choi W, Byon C. Internal natural convection around a sphere in a rectangular chamber. Int J Heat Mass Tran 2019;136:501–9.
[97] Pandey S, Park YG, Ha MY. An exhaustive review of studies on natural convection in enclosures with and without internal bodies of various shapes. Int J Heat Mass Tran 2019;138:762–95.
[98] Cho HW, Ha MY, Park YG. Natural convection in a square enclosure with two hot inner cylinders, Part II: the effect of two elliptical cylinders with various aspect ratios in a vertical array. Int J Heat Mass Tran 2019;135:962–73.
[99] Alomar OR, Basher NM, Yousif AA. Analysis of effects of thermal non-equilibrium and non-Darcy flow on natural convection in a square porous enclosure provided with a heated L shape plate. Int J Mech Sci 2020;181:105704.
[100] Dutta S, Kumar P, Kalita JC. Streamfunction-velocity computation of natural convection around heated bodies placed in a square enclosure. Int J Heat Mass Tran 2020;152:119550.
[101] Welhezi H, Ben-Cheikh N, Ben-Beya B. Numerical analysis of natural convection between a heated cube and its spherical enclosure. Int J Therm Sci 2020;150:105828.
[102] Boukendil M, El Moutaouakil L, Zrikem Z, Abdelbaki A. Coupled thermal radiation and natural convection heat transfer in a cavity with a discretely heated inner body. Mater Today Proc 2020;27:3065–70.
[103] Wang L, Huang C, Hu J, Shi B, Chai Z. Effects of temperature-dependent viscosity on natural convection in a porous cavity with a circular cylinder under local thermal non-equilibrium condition. Int J Therm Sci 2021;159:106570.
[104] Alomar OR, Basher NM, Yousif AA. Natural convection heat transfer from a bank of orthogonal heated plates embedded in a porous medium using LTNE model: a comparison between in-line and staggered arrangements. Int J Therm Sci 2021;160:106692.
[105] Zemach E, Spizzichino A, Feldman Y. Instability characteristics of a highly separated natural convection flow: configuration of a tandem of cold and hot horizontally oriented cylinders placed within a cold cubic enclosure. Int J Therm Sci 2021;159:106606.
[106] Wang Z, Wang T, Xi G, Huang Z. Periodic unsteady natural convection in square enclosure induced by inner circular cylinder with different vertical locations. Int Commun Heat Mass Tran 2021;124:105250.
[107] Vijaybabu TR. Influence of porous circular cylinder on MHD double-diffusive natural convection and entropy generation. Int J Mech Sci 2021;206:106625.
[108] Ikram MM, Saha G, Saha SC. Conjugate forced convection transient flow and heat transfer analysis in a hexagonal, partitioned, air filled cavity with dynamic modulator. Int J Heat Mass Tran 2021;167:120786.
[109] Ikram MM, Saha G, Saha SC. Unsteady conjugate heat transfer characteristics in hexagonal cavity equipped with a multi-blade dynamic modulator. Int J Heat Mass Tran 2023;200:123527.
[110] Saha S, Hussein AK, Saha G, Hussain S. Mixed convection in a tilted lid-driven square enclosure with adiabatic cylinder at the center. Int J Heat Technol 2011;29:143–56.
[111] Shankar PN, Deshpande MD. Fluid mechanics in the driven cavity. Annu Rev Fluid Mech 2000;32:93–136.
[112] Sparrow EM, Eichhorn R, Gregg JL. Combined forced and free convection in a boundary layer flow. Phys Fluids 1959;2:319–28.
[113] Shuja SZ, Yilbas BS, Iqbal MO. Mixed convection in a square cavity due to heat generating rectangular body. Effect of cavity exit port locations. Int J Numer Methods Heat Fluid Flow 2000;10:824–41.
[114] Rahman MM, Alim MA, Saha S, Chowdhury MK. A numerical study of mixed convection in a square cavity with a heat conducting square cylinder at different locations. J Mech Eng 2008;39:78–85.
[115] Oztop HF, Zhao Z, Yu B. Fluid flow due to combined convection in lid-driven enclosure having a circular body. Int J Heat Fluid Flow 2009;30:886–901.
[116] Hussain SH, Abd-Amer QR. Mixed convection heat transfer flow of air inside a sinusoidal corrugated cavity with a heat-conducting horizontal circular cylinder. J Enhanc Heat Transf 2011;18:433–47.
[117] Hussain SH, Hussein AK. Mixed convection heat transfer in a differentially heated square enclosure with a conductive rotating circular cylinder at different vertical locations. Int Commun Heat Mass Tran 2011;38:263–74.
[118] Islam AW, Sharif MAR, Carlson ES. Mixed convection in a lid driven square cavity with an isothermally heated square blockage inside. Int J Heat Mass Tran 2012;55:5244–55.
[119] Khanafer K, Aithal SM. Laminar mixed convection flow and heat transfer characteristics in a lid driven cavity with a circular cylinder. Int J Heat Mass Tran 2013;66:200–9.

[120] Ray S, Chatterjee D. MHD mixed convection in a lid-driven cavity including heat conducting circular solid object and corner heaters with Joule heating. Int Commun Heat Mass Tran 2014;57:200–7.
[121] Morshed KN, Sharif MAR, Islam AW. Laminar mixed convection in a lid-driven square cavity with two isothermally heated square internal blockages. Chem Eng Commun 2015;202:1176–90.
[122] Gangawane KM, Manikandan B. Mixed convection characteristics in lid-driven cavity containing heated triangular block. Chin J Chem Eng 2017;25:1381–94.
[123] Alsabery AI, Selimefendigil F, Hashim I, Chamkha AJ, Ghalambaz M. Fluid-structure interaction analysis of entropy generation and mixed convection inside a cavity with flexible right wall and heated rotating cylinder. Int J Heat Mass Tran 2019;140:331–45.
[124] Alsabery AI, Tayebi T, Roslan R, Chamkha AJ, Hashim I. Entropy generation and mixed convection flow inside a wavy-walled enclosure containing a rotating solid cylinder and a heat source. Entropy 2020;22:606.
[125] Cho HW, Park YG, Seo YM, Ha MY. Prediction of the heat transfer performance of mixed convection in a lid-driven enclosure with an elliptical cylinder using an artificial neural network. Numer Heat Tran 2020;78:29–47.
[126] Hamza MK, Mahdi QA. Investigation of mixed convection in cavity with different aspect ratio containing corrugated hot circular body. J Mech Eng Res Dev 2021; 44:251–64.
[127] Xiong PY, Hamid A, Iqbal K, Irfan M, Khan M. Numerical simulation of mixed convection flow and heat transfer in the lid-driven triangular cavity with different obstacle configurations. Int Commun Heat Mass Tran 2021;123:105202.
[128] Li Y, Zhou J, Tung S, Schneider E, Xi S. A review on development of nanofluid preparation and characterization. Powder Technol 2009;196:89–101.
[129] Dayf A, Feddadoui M, Bouchta S, Charef A, El Ihssini H. Effect of nanoparticles and base fluid types on natural convection in a three-dimensional cubic enclosure. Math Probl Eng 2021:2021.
[130] Yu ZT, Xu X, Hu YC, Fan LW, Cen KF. Unsteady natural convection heat transfer from a heated horizontal circular cylinder to its air-filled coaxial triangular enclosure. Int J Heat Mass Tran 2011;54:1563–71.
[131] Mahmoodi M, Sebdani SM. Natural convection in a square cavity containing a nanofluid and an adiabatic square block at the center. Superlattice Microst 2012; 52:261–75.
[132] Garoosi F, Jahanshaloo L, Rashidi MM, Badakhsh A, Ali ME. Numerical simulation of natural convection of the nanofluid in heat exchangers using a Buongiorno model. Appl Math Comput 2015;254:183–203.
[133] Ali O M, Kahwaji G Y. Numerical investigation of natural convection heat transferfrom square cylinder in an enclosed enclosure filled with nanofluids. Int J Recent Adv Mech Eng 2015;4:1–17.
[134] El Abdallaoui M, Hasnaoui M, Amahmid A. Numerical simulation of natural convection between a decentered triangular heating cylinder and a square outer cylinder filled with a pure fluid or a nanofluid using the lattice Boltzmann method. Powder Technol 2015;277:193–205.
[135] Sheikholeslami M, Ganji DD. Entropy generation of nanofluid in presence of magnetic field using Lattice Boltzmann Method. Phys A Stat Mech Its Appl 2015; 417:273–86.
[136] Boulahia Z, Sehaqui R. Numerical simulation of natural convection of nanofluid in a square cavity including a square heater. Int J Sci Res 2015;4:1718–22.
[137] Ravnik J, Škerget L. A numerical study of nanofluid natural convection in a cubic enclosure with a circular and an ellipsoidal cylinder. Int J Heat Mass Tran 2015; 89:596–605.
[138] Zhang T, Che D. Double MRT thermal lattice Boltzmann simulation for MHD natural convection of nanofluids in an inclined cavity with four square heat sources. Int J Heat Mass Tran 2016;94:87–100.
[139] Boulahia Z, Wakif A, Sehaqui R. Natural convection heat transfer of the nanofluids in a square enclosure with an inside cold obstacle. Int J Innov Sci Res 2016;21:367–75.
[140] Sheremet MA, Oztop HF, Pop I, Abu-Hamdeh N. Analysis of entropy generation in natural convection of nanofluid inside a square cavity having hot solid block: tiwari and das' model. Entropy 2016;18:9.
[141] Al-Rashed AAAA, Kalidasan K, Kolsi L, Borjini MN, Kanna PR. Three-dimensional natural convection of CNT-water nanofluid confined in an inclined enclosure with ahmed body. J Therm Sci Technol 2017;12. JTST0002–JTST0002.
[142] Al-Rashed AAAA, Kolsi L, Kalidasan K, Malekshah EH, Borjini MN, Kanna PR. Second law analysis of natural convection in a CNT-water nanofluid filled inclined 3D cavity with incorporated Ahmed body. Int J Mech Sci 2017;130: 399–415.
[143] Dogonchi AS, Sheremet MA, Pop I, Ganji DD. MHD natural convection of Cu/H2O nanofluid in a horizontal semi-cylinder with a local triangular heater. Int J Numer Methods Heat Fluid Flow 2018;28:2979–96.
[144] Rahimi A, Kasaeipoor A, Malekshah EH, Amiri A. Natural convection analysis employing entropy generation and heatline visualization in a hollow L-shaped cavity filled with nanofluid using lattice Boltzmann method- experimental thermo-physical properties. Phys E Low-Dimensional Syst Nanostructures 2018; 97:82–97.
[145] Rahimi A, Sepehr M, Lariche MJ, Mesbah M, Kasaeipoor A, Malekshah EH. Analysis of natural convection in nanofluid-filled H-shaped cavity by entropy generation and heatline visualization using lattice Boltzmann method. Phys E Low-Dimensional Syst Nanostructures 2018;97:347–62.
[146] Roy NC. Natural convection of nanofluids in a square enclosure with different shapes of inner geometry. Phys Fluids 2018;30:113605.
[147] Aly AM, Raizah ZAS. Incompressible smoothed particle hydrodynamics (ISPH) method for natural convection in a nanofluid-filled cavity including rotating solid structures. Int J Mech Sci 2018;146–147:125–40.
[148] KhakRah HR, Mohammaei M, Hooshmand P, Bagheri N, Hasani Malekshah E. Free convection analysis in a Γ-shaped heat exchanger using lattice Boltzmann method employing second law analysis and heatline visualization. Int J Numer Methods Heat Fluid Flow 2019;29:3056–74.
[149] Hussain SH, Rahomey MS. Comparison of natural convection around a circular cylinder with different geometries of cylinders inside a square enclosure filled with Ag nanofluid superposed porous nanofluid layers. J Heat Tran 2019:141.
[150] Tayebi T, Chamkha AJ, Djezzar M. Natural convection of CNT-water nanofluid in an annular space between confocal elliptic cylinders with constant heat flux on inner wall. Sci Iran 2019;26:2770–83.
[151] Majdi HS, Abdulkadhim A, Abed AM. Numerical investigation of natural convection heat transfer in a parallelogramic enclosure having an inner circular cylinder using liquid nanofluid. Front Heat Mass Transf 2019;12.
[152] Abdulkadhim A, Hamzah HK, Ali FH, Abed AM, Abed IM. Natural convection among inner corrugated cylinders inside wavy enclosure filled with nanofluid superposed in porous–nanofluid layers. Int Commun Heat Mass Tran 2019;109: 104350.
[153] Vijaybabu TR, Dhinakaran S. MHD Natural convection around a permeable triangular cylinder inside a square enclosure filled with Al 2 O 3 –H 2 O nanofluid: an LBM study. Int J Mech Sci 2019;153–154:500–16.
[154] Tayebi T, Chamkha AJ. Entropy generation analysis during MHD natural convection flow of hybrid nanofluid in a square cavity containing a corrugated conducting block. Int J Numer Methods Heat Fluid Flow 2020;30:1115–36.
[155] Alsabery AI, Alsabery AI, Hashim I, Hajjar A, Pour MS. Entropy generation and natural convection flow of hybrid nanofluids in a partially divided wavy cavity including solid blocks. Energies 2020;13:2942.
[156] Tayebi T, Öztop HF, Chamkha AJ. Natural convection and entropy production in hybrid nanofluid filled-annular elliptical cavity with internal heat generation or absorption. Therm Sci Eng Prog 2020;19:100605.
[157] Dogonchi AS, Tayebi T, Chamkha AJ, Ganji DD. Natural convection analysis in a square enclosure with a wavy circular heater under magnetic field and nanoparticles. J Therm Anal Calorim 2020;139:661–71.
[158] Moria H. Natural convection in an L-shape cavity equipped with heating blocks and porous layers. Int Commun Heat Mass Tran 2021;126:105375.
[159] Patpatiya P, Pandey I, Gupta A. Comparison of free convection around heated square plate in a square enclosure enclosing air, water, and Ag-water nanofluid. Lect Notes Mech Eng, Springer Science and Business Media Deutschland GmbH; 2021. p. 141–50.
[160] Abdulkadhim A, Hamzah HK, Ali FH, Yıldız Ç, Abed AM, Abed EM, et al. Effect of heat generation and heat absorption on natural convection of Cu-water nanofluid in a wavy enclosure under magnetic field. Int Commun Heat Mass Tran 2021;120: 105024.
[161] Hosseinjani AA, Roohi AH. Immersed boundary method for MHD unsteady natural convection around a hot elliptical cylinder in a cold rhombus enclosure filled with a nanofluid. SN Appl Sci 2021;3:1–25.
[162] Aly AM, Raizah ZAS, Ahmed SE. ISPH simulations of natural convection from rotating circular cylinders inside a horizontal wavy cavity filled with a nanofluid and saturated by a heterogeneous porous medium. Eur Phys J Spec Top 2021;230: 1173–83.
[163] Mirzaie M, Lakzian E. Natural convection of nanofluid-filled annulus with cooled and heated sources and rotating cylinder in the water near the density inversion point. Eur Phys J Plus 2021;136:1–20.
[164] Munawar S, Saleem N, Khan WA. Entropy generation minimization E.G.M. analysis of free convective hybrid nanofluid flow in a corrugated triangular annulus with a central triangular heater. Chin J Phys 2022;75:38–54.
[165] Kalteh M, Javaherdeh K, Azarbarzin T. Numerical solution of nanofluid mixed convection heat transfer in a lid-driven square cavity with a triangular heat source. Powder Technol 2014;253:780–8.
[166] Chatterjee D, Gupta SK, Mondal B. Mixed convective transport in a lid-driven cavity containing a nanofluid and a rotating circular cylinder at the center. Int Commun Heat Mass Tran 2014;56:71–8.
[167] Selimefendigil F, Öztop HF. Numerical study of MHD mixed convection in a nanofluid filled lid driven square enclosure with a rotating cylinder. Int J Heat Mass Tran 2014;78:741–54.
[168] Boulahia Z, Wakif A, Sehaqui R. Numerical study of mixed convection of the nanofluids in two-sided lid-driven square cavity with a pair of triangular heating cylinders. J Eng 2016;2016.
[169] Alsabery AI, Ismael MA, Chamkha AJ, Hashim I. Numerical investigation of mixed convection and entropy generation in a wavy-walled cavity filled with nanofluid and involving a rotating cylinder. Entropy 2018;20:664.
[170] Boulahia Z, Wakif A, Sehaqui R. Heat transfer and cu-water nanofluid flow in a ventilated cavity having central cooling cylinder and heated from the below considering three different outlet port locations. Front Heat Mass Transf 2018;11.
[171] Hussein AK, Hamzah HK, Ali FH, Kolsi L. Mixed convection in a trapezoidal enclosure filled with two layers of nanofluid and porous media with a rotating circular cylinder and a sinusoidal bottom wall. J Therm Anal Calorim 2020;141: 2061–79.
[172] Thakur MSH, Islam M, Karim AU, Saha S, Hasan MN. Numerical study of laminar mixed convection in a Cu-water nanofluid filled lid-driven square cavity with an isothermally heated cylinder. AIP Conf Proc 2019;2121:70017. American Institute of Physics Inc.
[173] Jamal M, Hussain S. Mixed convection in square enclosure by considering the thermal effect on cylinder. J Thermophys Heat Tran 2021;35:869–82.
[174] Ishak MS, Alsabery AI, Hashim I, Chamkha AJ. Entropy production and mixed convection within trapezoidal cavity having nanofluids and localised solid cylinder. Sci Rep 2021;11:1–22.

[175] Ali MM, Akhter R, Alim MA. Hydromagnetic mixed convection in a triangular shed filled by nanofluid and equipped with rectangular heater and rotating cylinders. Int J Thermofluids 2021;11:100105.
[176] Shirani N, Toghraie D. Numerical investigation of transient mixed convection of nanofluid in a cavity with non-Darcy porous inner block and rotating cylinders with harmonic motion. Sci Rep 2021;11:1–18.
[177] Buongiorno J. Convective transport in nanofluids. J Heat Tran 2006;128:240–50.
[178] Malvandi A, Ganji DD. Effects of nanoparticle migration on water/alumina nanofluid flow inside a horizontal annulus with a moving core. J Mech 2015;31: 291–305.
[179] Alsabery AI, Ismael MA, Chamkha AJ, Hashim I. Mixed convection of Al2O3-water nanofluid in a double lid-driven square cavity with a solid inner insert using Buongiorno's two-phase model. Int J Heat Mass Tran 2018;119:939–61.
[180] Hashim I, Alsabery AI, Sheremet MA, Chamkha AJ. Numerical investigation of natural convection of Al2O3-water nanofluid in a wavy cavity with conductive inner block using Buongiorno's two-phase model. Adv Powder Technol 2019;30: 399–414.
[181] Alsabery AI, Gedik E, Chamkha AJ, Hashim I. Effects of two-phase nanofluid model and localized heat source/sink on natural convection in a square cavity with a solid circular cylinder. Comput Methods Appl Mech Eng 2019;346:952–81.
[182] Alsabery AI, Gedik E, Chamkha AJ, Hashim I. Impacts of heated rotating inner cylinder and two-phase nanofluid model on entropy generation and mixed convection in a square cavity. Heat Mass Transf Und Stoffuebertragung 2020;56: 321–38.
[183] Barnoon P, Toghraie D, Dehkordi RB, Abed H. MHD mixed convection and entropy generation in a lid-driven cavity with rotating cylinders filled by a nanofluid using two phase mixture model. J Magn Magn Mater 2019;483:224–48.
[184] Alsabery AI, Armaghani T, Chamkha AJ, Hashim I. Two-phase nanofluid model and magnetic field effects on mixed convection in a lid-driven cavity containing heated triangular wall. Alex Eng J 2020;59:129–48.
[185] Hashem Zadeh SM, Sabour M, Sazgara S, Ghalambaz M. Free convection flow and heat transfer of nanofluids in a cavity with conjugate solid triangular blocks: employing Buongiorno's mathematical model. Phys A Stat Mech Its Appl 2020; 538:122826.
[186] Buongiorno J, Venerus DC, Prabhat N, McKrell T, Townsend J, Christianson R, et al. A benchmark study on the thermal conductivity of nanofluids. J Appl Phys 2009;106:94312.
[187] Alsabery AI, Ghalambaz M, Armaghani T, Chamkha A, Hashim I, Pour MS. Role of rotating cylinder toward mixed convection inside a wavy heated cavity via two-phase nanofluid concept. Nanomaterials 2020;10:1–22.
[188] Alhashash A. Natural convection of nanoliquid from a cylinder in square porous enclosure using Buongiorno's two-phase model. Sci Rep 2020;10:1–12.
[189] Al-Kouz W, Bendrer BAI, Aissa A, Almuhtady A, Jamshed W, Nisar KS, et al. Galerkin finite element analysis of magneto two-phase nanofluid flowing in double wavy enclosure comprehending an adiabatic rotating cylinder. Sci Rep 2021;11:1–15.