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Buckling Analysis of Cantilever Nanoactuators Immersed in an Electrolyte: A Close Form Solution Using Duan-Rach Modified Adomian Decomposition Method

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Abstract

A new modified Adomian Decomposition Method (ADM) was utilized to obtain an analytical solution for the buckling of the nanocantilever actuators immersed in liquid electrolytes. The nanoactuators in electrolytes are subject to different nonlinear forces including ionic concentration, van der Waals, external voltage and electrochemical forces. The Duan–Rach modified Adomian decomposition method was used to obtain a full explicate solution for the buckling of nanoactuators free of any undetermined coefficients. The results were compared with those of Wazwas ADM and of a finite element method available in the literature and excellent agreement was found between them.

Keywords: Nanoactuator; Duan and Rach ADM; Analytic Solution; Electrolyte.

1. Introduction

Micro/nano beams actuators play a crucial role in micro/nano technology. Nano beam actuators have been increasingly utilized in many different applications. For instance, they have been utilized as sensors for gas detection [1] and force measurement [2]. They have also been used as gravimetric sensors [3] and biomolecular fingerprinting sensors [4]. Furthermore, these actuators are suitable potential candidates for nano-electro-mechanical nonvolatile memories [5] and switches [6].

In biomedical applications, micro/nano cantilever beams actuators are extensively employed as embedded devices or sensors in electrolyte liquids [7-9]. An electrolyte induces nonlinear forces, e.g., ionic and chemical forces, on the actuator. As a result, the presence of the electrolyte increases the complexity of the nanoactuator deflection behavior. In many applications of nanoactuators, as sensors, the buckling and deflection shape of an actuator beam is of considerable importance. In a very recent study, Noghrehabadi et al. [8] studied the cantilever nanactuators in liquid electrolytes, finding that the governing differential equation, representing the behavior of the beam subject to the applied forces, is fourth order nonlinear boundary value. Noghrehabadi et al. [8] applied the modified Adomian Decomposition Method (ADM), proposed by Wazwaz [10], to obtain an analytical solution for the cantilever beam nanoactuators. The ADM is naturally an Initial Value (IV) method. As the governing equation of the cantilever beam is a boundary value differential equation, the researchers used the method of undetermined coefficients to deal with the problem. Hence, they transformed the boundary value differential equation of the nanobeam into an IV differential equation, containing two undetermined initial values (C_1 and C_2). Therefore, the obtained analytical solution contains unknown constants, which should be evaluated later by use of the remaining unutilized boundary conditions. However, finding the undetermined coefficients by using the remaining boundary conditions requires that a set of highly nonlinear algebraic equations be solved, which demands numerical procedures. Hence, the presence of the undetermined constants in the analytic solution significantly reduces the explicitness of the solution and changes the solution to an implicit one. In addition, solving a set of highly nonlinear algebraic equations for undetermined coefficients, i.e. C_1 and C_2, leads to different sets of roots. Finding and choosing the correct set of roots is another problem, which arises with the method of undetermined coefficients. The number of possible roots tediously increases as the size of the Adomian series increases. The Adomian decomposition method with undetermined coefficients has been utilized in many previous studies of fluid mechanics problems [11] as well as nanactuators [12-17]; hence, the effectiveness of this method has been demonstrated.

Recently, Duan and Rach [18] proposed a new Adomian decomposition method for high-order boundary-value problems. The idea is to utilize all of the boundary conditions to derive an integral equation before establishing the recursion scheme for the solution components. Thus, a modified recursion scheme can be derived without any undetermined coefficients when computing successive solution components. The Duan and Rach ADM can be also applicable for ribbon boundary conditions as discussed by Duan et al. [19].

In the present study, the modified Adomian decomposition method, proposed by Duan and Rach [18], is utilized to obtain an analytical solution for bucking of nano beam actuators immersed in liquid electrolytes. The obtained analytical solution is free of any undetermined coefficients. Hence, it could be directly utilized for further design and measuring applications of nano cantilever actuators.

2. The Mathematical Governing Equation:

According to Noghrehabadi et al.'s [8] study, the governing equation of a nanobeam actuator immersed in a liquid electrolyte, is written as:

$$\frac{d^4u}{dx^4} = -\frac{\alpha}{(1+\delta)u^3} + \frac{\beta}{(1+\delta)\sinh^2(\xi_0 u)}\left\{\frac{\phi_2}{\phi_1}\cosh(\xi_0 u) - \frac{1}{2}\left[1+\left(\frac{\phi_2}{\phi_1}\right)^2\right]\right\} \tag{1}$$

where u is the non-dimensional deflection of the beam, and x is the non-dimensional position along the beam measured from the supported end. In Eq. (1), α, β, ϕ_2/ϕ_1, ξ_0 and δ are non-dimensional parameters, which denote the van der Waals attractions, the electrochemical force, the voltage ratio, the ionic concentration and the size effect, respectively. Parameters of α and δ are related to the nanosize effects. The detailed description of the geometrical and physical parameters is presented in Noghrehabadi et al.'s [8] work. As the beam is cantilever, one end is supported and the other one is free (natural). Therefore, the boundary conditions for a cantilever beam are written as [8]:

$$u(0)=1,\ u'(0)=0,\ u''(1)=u'''(1)=0 \tag{2}$$

For some nanactuators, there is a support in the bending side, which acts as a torsional spring. Hence, the boundary conditions at the supported and free ends for a simply supported (SS) nanoactuator can be written as [8]:

$$u(0)=1,\ u''(0)=ku'(0),\ u''(1)=u'''(1)=0 \tag{3}$$

where k is the non-dimensional parameter for the stiffness of the supported end. For very large values of the stiffness parameter, the support acts as a rigid wall, and hence, the boundary condition of $u''=ku'(0)$ reduces to the regular boundary condition of $u'(0)=0$.

3. Solution method

Here, the modified Adomian decomposition method, proposed by Duan and Rach [18], is applied to obtain an analytic solution for the nonlinear differential equation of the nanoactuator (Eq. (1)) subject to the prescribed boundary conditions (Eqs. (2) or (3)). This section falls into three parts; in the first part, Eq. (1) will be solved subject to the boundary conditions of Eq. (2) as a common special case. In the second part, Eq. (1) will be solved subject to the boundary conditions of Eq. (3), as a more general case. Finally, the conventional method of undetermined coefficients will be recalled, and the advantages of the present study will be discussed in the third part.

Based on the Adomian decomposition method, the differential equation, Eq. (1), in operator form is written as [18]:

$$Lu = Nu, \qquad 0<x<1 \tag{4}$$

where $L(.)=d^4/dx^4$ is the linear differential operator to be inverted, and Nu is an analytic nonlinear operator. Here, Nu is the nonlinear part of the differential equation written as:

$$Nu = -\frac{\alpha}{(1+\delta)u^3} + \frac{\beta}{(1+\delta)\sinh^2(\xi_0 u)}\left\{\frac{\phi_2}{\phi_1}\cosh(\xi_0 u) - \frac{1}{2}\left[1+\left(\frac{\phi_2}{\phi_1}\right)^2\right]\right\} \tag{5}$$

L^{-1} is the inverse linear operator.

3.1 Cantilever nanoactuator

The governing equation of a cantilever beam immersed in a liquid electrolyte is Eq. (1) which should be solved subject to the boundary conditions of Eq. (2). It can be inferred from the boundary conditions in Eq. (2) that $u(0)$, $u'(0)$, $u''(1)$ and $u'''(1)$ correspond to function, its first, second and third derivatives, respectively. Hence, based on the new modified Adomian decomposition method [18], the inverse operator, L^{-1}, is selected as:

$$L^{-1}(.) = \int_0^x\int_0^x\int_1^x\int_1^x (.)\, dxdxdxdx \tag{6}$$

Applying the introduced invers operator to the linear part of the governing equation, Lu, yields:

$$L^{-1}Lu = \int_0^x\int_0^x\int_1^x\int_1^x \left(u^{(4)}(x)\right) dxdxdxdx = u(x) - u(0) - xu'(0) - \frac{x^2}{2}u''(1) - \left(\frac{x^3}{6} - \frac{x^2}{2}\right)u'''(1) \tag{7}$$

Using the boundary conditions of Eq. (2) results in $L^{-1}Lu=u(x)-1$. Applying the inverse operator to the both sides of Eq. (4) gives:

$$u(x) - 1 = L^{-1}Nu \tag{8}$$

It is clear that all the boundary conditions were utilized by selecting the inverse operator in the form of Eq. (6); hence, there is not any remaining undetermined coefficient in Eq. (8). Now, following the Adomian method, $u(x)$ and the nonlinearity term of Nu are decomposed to series of Adomian polynomials as:

$$u(x) = \sum_{m=0}^{\infty} u_m(x), \qquad Nu = \sum_{m=0}^{\infty} A_m(x) \tag{9}$$

where $A_m = A_m\,(u_0(x), u_1(x),\ \ldots\ , u_m(x))$ are the Adomian polynomials which can be evaluated by means of the Adomian definition formula as [20]:

$$A_m = \frac{1}{m!}\frac{d^m}{d\lambda^m} N\left(\sum_{k=0}^{\infty} u_k \lambda^k\right)\Bigg|_{\lambda=0}, \qquad m \geq 0, \tag{10}$$

Substituting the Adomian polynomials in Eq. (8) results in:

$$\sum_{m=0}^{\infty} u_m(x) = 1 + L^{-1}\sum_{m=0}^{\infty} A_m(x) \tag{11}$$

As the nonlinearity term (Nu) is only a function of u (i.e., assuming $Nu=f(u)$), the five first Adomian polynomials are easily evaluated as:

$$
\begin{aligned}
A_0 &= f(u_0),\\
A_1 &= f'(u_0)u_1,\\
A_2 &= f'(u_0)u_2 + f''(u_0)\frac{u_1^2}{2!},\\
A_3 &= f'(u_0)u_3 + f''(u_0)u_2 + f'''(u_0)\frac{u_1^3}{3!},\\
A_4 &= f'(u_0)u_4 + f''(u_0)\left(\frac{u_2^2}{2!} + u_1u_3\right) + f'''(u_0)\frac{u_1^2u_2}{2!} + f^{(4)}u_0\frac{u_1^4}{4!},
\end{aligned}
\tag{12}
$$

where the nonlinearity function was introduced in Eq. (5). Using Eq. (12), we evaluate the three first terms of A_m as:

$$
A_0 = -\frac{\alpha}{(1+\delta)u_0^3} + \frac{\beta}{(1+\delta)\sinh^2(\xi_0 u_0)}\left\{\frac{\phi_2}{\phi_1}\cosh(\xi_0 u_0) - \frac{1}{2}\left[1+\left(\frac{\phi_2}{\phi_1}\right)^2\right]\right\}
\tag{12a}
$$

$$
A_1 = u_1\left(\frac{\beta\zeta_0\frac{\phi_2}{\phi_1}}{(1+\delta)\sinh(\xi_0 u_0)} - \frac{3\alpha}{(1+\delta)u_0^4} - \frac{2\beta\zeta_0\cosh(\zeta u_0)\left(\frac{\phi_2}{\phi_1}\cosh(\zeta_0 u_0) - \frac{1}{2}\left(1+\left(\frac{\phi_2}{\phi_1}\right)^2\right)\right)}{(1+\delta)\sinh^3(\xi_0 u_0)}\right)
\tag{12b}
$$

$$
\begin{aligned}
A_2 = &-\frac{3u_2\alpha}{(1+\delta)u_0^4} + \frac{u_2\beta\zeta_0\frac{\phi_2}{\phi_1}}{(1+\delta)\sinh(\xi_0 u_0)}\\
&-\frac{2u_2\beta\zeta_0\cosh(\zeta u_0)\left(\frac{\phi_2}{\phi_1}\cosh(\zeta_0 u_0) - \frac{1}{2}\left(1+\left(\frac{\phi_2}{\phi_1}\right)^2\right)\right)}{(1+\delta)\sinh^3(\xi_0 u_0)}\\
&-\frac{6u_1^2\alpha}{(1+\delta)u_0^5} - \frac{3u_1^2\beta\zeta_0^2\frac{\phi_2}{\phi_1}\cosh(\zeta_0 u_0)}{2(1+\delta)\sinh^2(\xi_0 u_0)}\\
&+\frac{3u_1^2\beta\zeta_0^2\cosh^2(\zeta u_0)\left(\frac{\phi_2}{\phi_1}\cosh(\zeta_0 u_0) - \frac{1}{2}\left(1+\left(\frac{\phi_2}{\phi_1}\right)^2\right)\right)}{(1+\delta)\sinh^3(\xi_0 u_0)}\\
&-\frac{u_1^2\beta\zeta_0^2\left(\frac{\phi_2}{\phi_1}\cosh(\zeta_0 u_0) - \frac{1}{2}\left(1+\left(\frac{\phi_2}{\phi_1}\right)^2\right)\right)}{(1+\delta)\sinh^2(\xi_0 u_0)}
\end{aligned}
\tag{12c}
$$

In order to establish a recursion scheme, we use a new convergence parameter, known as Duan's convergence parameter [18, 21]. Embedding Duan's parameter, c, in the definition of the zeroth-order and first-order solution components, *e.g.* u_0 and u_1, can significantly modify the solution's convergence properties. Once the Duan's parameter, c, embedded and Eq. (11) is used, the recursion scheme is established as:

$$\begin{aligned} &u_0(x)=c,\\ &u_1(x)=1-c+L^{-1}A_0(x)\\ &u_{m+1}(x)=L^{-1}A_m(x),\quad m\geq 0 \end{aligned} \tag{13}$$

The first three Adomian stages are obtained as:

$$u_0(x)=c, \tag{14a}$$

$$u_1(x)=1-c+\frac{1}{4}h_1x^2-\frac{1}{6}h_1x^3+\frac{1}{24}h_1x^4 \tag{14b}$$

$$u_2(x)=-\frac{1}{24}h_2x^2\left(-\frac{1}{1680}h_1x^6+\frac{1}{210}h_1x^5-\frac{1}{60}h_1x^4+(-1+c)x^2+\left(4+\frac{1}{5}h_1-4c\right)x-\frac{13}{30}h_1+6c-6\right) \tag{14c}$$

$$\begin{aligned} u_3(x)=&\left[\frac{1}{24}h_4\left(\frac{73}{1890}h_1^2+\left(-\frac{13}{15}c+\frac{13}{15}\right)h_1+6\left(-1+c^2\right)\right)+\frac{13}{30}h_3\left(-1-\frac{661}{8190}h_1+c\right)\right]x^2\\ &+\left[\frac{1}{24}h_4\left(-\frac{13}{810}h_1^2+\frac{2}{5}(c-1)h_1-4(c-1)^2\right)-\frac{1}{5}h_3\left(c-1-\frac{13}{162}h_1\right)\right]x^3+\frac{1}{24}h_4(c-1)^2x^4\\ &+\left(-\frac{1}{720}h_4(c-1)h_1+\frac{1}{60}h_3\left(c-1-\frac{13}{180}h_1\right)\right)x^6+\left(\frac{1}{2520}h_4h_1(c-1)-\frac{1}{210}h_3\left(c-1-\frac{1}{20}h_1\right)\right)x^7\\ &+\left(-\frac{1}{20160}h_1h_4\left(c-1-\frac{3}{4}h_1\right)+\frac{1}{1680}h_3(c-1)\right)x^8-\frac{1}{36288}h_1^2h_4x^9+\frac{1}{103680}h_1\left(h_1h_4-\frac{12}{35}h_3\right)x^{10}\\ &-\frac{1}{570240}h_1\left(h_1h_4-\frac{12}{35}h_3\right)x^{11}+\frac{1}{6842880}h_1\left(h_1h_4-\frac{12}{35}h_3\right)x^{12} \end{aligned} \tag{14d}$$

where the parameters of h_1-h_4 are facilitated to reduce the manipulation calculations and the representation size of the series. The values of h_1-h_4 are solely functions of non-dimensional parameters of α, β, ξ_0, δ, ϕ_2/ϕ_1 and Duan's parameter c.

$$h_1=-\frac{\alpha}{(1+\delta)c^3}-\frac{\beta\left(1-2\frac{\phi_1}{\phi_2}\cosh(c\zeta_0)+\left(\frac{\phi_1}{\phi_2}\right)^2\right)}{2(1+\delta)\sinh^2(c\zeta_0)} \tag{15a}$$

$$h_2=\frac{3\alpha}{(1+\delta)c^4}+\frac{\beta\zeta_0\frac{\phi_1}{\phi_2}}{\sinh(c\zeta_0)(1+\delta)}-\frac{2\beta\left(\frac{\phi_1}{\phi_2}\cosh(c\zeta_0)-\frac{1}{2}-\frac{1}{2}\left(\frac{\phi_1}{\phi_2}\right)^2\right)\cosh(c\zeta_0)\zeta_0}{(1+\delta)\sinh^3(c\zeta_0)} \tag{15b}$$

$$h_3=h_2\frac{\frac{2}{3}\beta\zeta_0c^4\frac{\phi_1}{\phi_2}\cosh^2(c\zeta_0)-\frac{1}{3}\beta\zeta_0c^4\left(1+\left(\frac{\phi_1}{\phi_2}\right)^2\right)\cosh(c\zeta_0)-\left(\alpha\sinh(c\zeta_0)+\frac{1}{3}\beta\zeta c^4\frac{\phi_1}{\phi_2}\right)\sinh^2(c\zeta_0)}{8(1+\delta)c^4\sinh^3(c\zeta_0)} \tag{15c}$$

$$h_4 = \frac{12\alpha\sinh^4(c\zeta_0) + \beta c^5\zeta_0^2\left(-5\frac{\phi_1}{\phi_2}\cosh(c\zeta_0) + 1 + \left(\frac{\phi_1}{\phi_2}\right)^2\right)\sinh^2(c\zeta_0)}{2(1+\delta)c^5\sinh^4(c\zeta_0)} - \frac{-3\beta c^5\zeta_0^2\cosh^2(c\zeta_0)\left(-2\frac{\phi_1}{\phi_2}\cosh(c\zeta_0) + 1 + \left(\frac{\phi_1}{\phi_2}\right)^2\right)}{2(1+\delta)c^5\sinh^4(c\zeta_0)} \tag{15d}$$

Finally, the *m-th* stage solution approximants is evaluated as $\phi_m(x)=u_0+u_1+u_2+u_3+\ldots u_m$. It is worth noting that assuming c=1 eliminates the effect of Duan's parameter and reduces the present solution to the results of the Duan-Rach modified Adomian decomposition method without using Duan's parameter. Since there is not any exact solution available for the deflection of the nanoactuator, the error of the solution can be evaluated by using its reminder. The reminder $R(x)$ can be evaluated by substituting the approximate solution ($\phi_m(x)$) in the governing equation:

$$R(x) = \frac{d^4\phi_m}{dx^4} + \frac{\alpha}{(1+\delta)\phi_m^3} - \frac{\beta}{(1+\delta)\sinh^2(\xi_0\phi_m)}\left\{\frac{\phi_2}{\phi_1}\cosh(\xi_0\phi_m) - \frac{1}{2}\left[1 + \left(\frac{\phi_2}{\phi_1}\right)^2\right]\right\} \tag{16}$$

If ϕ_m is the exact solution, the reminder $R(x)$ is zero. The non-zero values of $R(x)$ show the error of the approximation solution. The maximum error can be evaluated using:

$$MaxErr = Max\left|R(x)\right| \tag{17}$$

Considering a typical case of nanoactuator with α=1,β=1, ζ_0=1, ϕ_2/ϕ_1=0.1 and δ=0.5, adopted by Noghrehabadi et al. [8], we evaluate and plot the values of $R(x)$ and *MaxErr* for different values of Duan's parameter c in Figs. 1 and 2, respectively. Fig. 1 shows that c=0.9 provides less reminder error over the length of the beam. Fig. 2, confirming Fig. 1, shows that for ϕ_2 there is an optimum value of c around 0.92. It is interesting that the case of ϕ_2 (using the optimum value of c in) can provide the same reminder error as that of ϕ_3 and ϕ_4 (without using Duan's constant (i.e. c=1)).

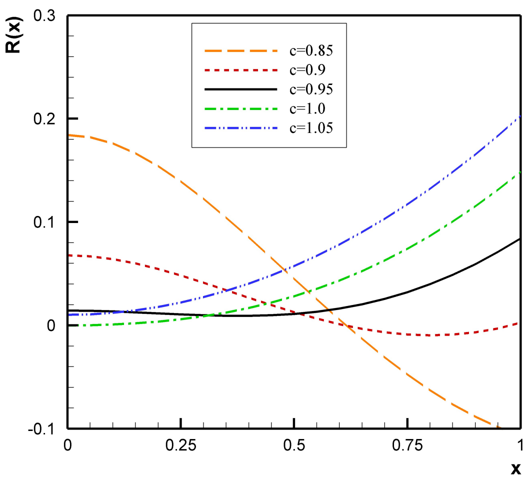

Fig. 1. Variation of the error reminder ($R(x)$) over the length of the nanoactuator for selected values of Duan's parameter for $\phi_2(x)$ when α=1, β=1, ζ_0=1, ϕ_2/ϕ_1=0.1 and δ=0.5.

A comparison between the evaluated tip deflections of the nanoactuator (i.e. $u_{tip}=u(1)$) using the results of the present study and the results of a Finite Element Method (FEM) as well as a ADM [8] is performed in Table 1. Furthermore, the tip deflection of the nanoactuator for the selected values of c and different stages of the Adomian series is shown in Table 1. The results of this table show that the accuracy of the solution increases as the number of stages increases. In agreement with the results of Figs. 1 and 2, the results of Table 1 show that for c around 0.9 the difference between the evaluated tip deflection using present method and results of FDM is very small. Indeed, the tip deflection evaluated using ϕ_2 with c=0.9 is more accurate than that of ϕ_5 with c=1 (i.e., c=1 represents Duan-Rach-ADM without using Duan's parameter).

Table 1. Tip deflection of a cantilever nanoactuator when α=1, β=1, ζ_0=1, ϕ_2/ϕ_1=0.1 and δ=0.5.

c	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5
0.85	0.16912	0.14625	0.14151	0.14004	0.13994
0.9	0.14292	0.13996	0.14016	0.14014	0.14015
0.95	0.12176	0.13312	0.13725	0.13885	0.13953
1.0	0.10449	0.12591	0.13347	0.13674	0.13832
1.05	0.09025	0.11853	0.12921	0.13413	0.13666
ADM [8]	-	-	-	0.13916	0.14008
FEM [8]			0.14014		

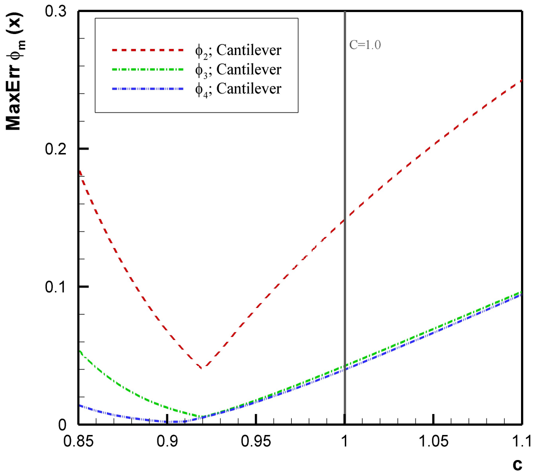

Fig. 2. Maximum value of reminder (MaxErr) as a function of Duan's parameter (c) for different stages of Adomian polynomials when α=1, β=1, ζ_0=1, ϕ_2/ϕ_1=0.1 and δ=0.5.

It is clear that the presence of an appropriate value of Duan's parameter would significantly increase the convergence rate of the series solution. The Duan's parameter is very useful in increasing the resolution of the solution with few stages of the Adomian polynomials to make a very compact robust solution for a specified nanoactuator.

3.2 Simply Supported (SS) nanoactuator

The boundary conditions in the case of simply supported nanoactuator are represented by Eq. (3). As can be seen in this case, the second derivative of the deflection is related to the slope of the beam through the non-dimensional spring constant k at the bending end (*i.e.*, $u''(0)=ku'(0)$). This is an implicit boundary condition which has been discussed neither in the original method of new Duan-Rach ADM [18] nor in the extended work of Duan et al. [19]. However, the undetermined boundary conditions should be evaluated by using the idea of the Duan-Rach ADM [18] and all of the boundary conditions and introducing a proper inverse operator. Afterwards, the recursive scheme can be initiated. In Eq. (3), two of the boundary conditions are given in the second and third derivatives at the natural end (i.e. $u''(1)=0$ and $u'''(1)=0$), another boundary condition is related to the function at the bending end (i.e. $u(0)=1$), and the remaining boundary condition relates to the first and second derivatives at the bending end ($u''(0)=ku'(0)$). Hence, the Adomian operator L^{-1} is introduced in the same manner as the cantilever case in Eq. (6). Applying the inverse operator to the linear part of the governing equation (Eq. (1)) yields the same equation as Eq. (7). Now, applying the inverse operator on the both sides of Eq. (4) and using the three available explicit boundary conditions (i.e. $u''(1)=0$, $u'''(0)=0$ and $u(0)=1$), we get:

$$u(x)-1-xu'(0)=L^{-1}Nu \tag{18}$$

As there is not any available explicit boundary condition for $u'(0)$ to be substituted in Eq. (8), we try to find this boundary condition with the aid of the remaining unused boundary condition (i.e. $u''(0)=k.u'(0)$). Differentiating Eq. (8) twice yields:

$$u''(x)=\int_1^x\int_1^x Nu\ dxdx \tag{19}$$

where substituting x=0 leads to:

$$u''(0)=\int_1^0\int_1^x Nu\ dxdx \tag{20}$$

Next, using the remaining boundary condition, (i.e., $u''(0)=ku'(0)$) and Eq. (20), we obtain the unknown value of $u'(0)$ as:

$$u'(0)=\frac{1}{k}\int_1^0\int_1^x Nu\ dxdx \tag{21}$$

substituting Eq. (21) for Eq. (18) yields:

$$u(x)=1+x\frac{1}{k}\int_1^0\int_1^x Nu\ dxdx+L^{-1}Nu \tag{22}$$

As can be seen above, there is not any remaining undetermined coefficient in Eq. (8). Using Eq. (8) and utilizing the Duan's parameter, we establish the recursive scheme as:

$$\begin{aligned}
&u_0(x)=c,\\
&u_1(x)=1-c+x\frac{1}{k}\int_1^0\int_1^x A_0(x)\ dxdx+L^{-1}A_0(x)\\
&u_{m+1}(x)=x\frac{1}{k}\int_1^0\int_1^x A_m(x)\ dxdx+L^{-1}A_m(x),\qquad m\geq 0
\end{aligned} \tag{23}$$

Applying the recursive scheme and utilizing Eqs. (12) for nonlinearity terms, we obtain the Adomian polynomials of Eq. (1) subject to boundary conditions of Eq. (3) as follows:

$$u_0(x)=c, \tag{24a}$$

$$u_1(x)=1-c+\frac{q_1}{2k}x+\frac{q_1}{4}x^2-\frac{q_1}{6}x^3+\frac{q_1}{24}x^4, \tag{24b}$$

$$\begin{aligned}
u_2(x)=&-\frac{1}{360k}q_2\left(60q_1+180k(1-c)+13kq_1\right)x-\frac{1}{40320}q_2\left(3360q_1+10080k(1-c)+728kq_1\right)x^2\\
&+\frac{1}{40320}q_2\left(1680q_1+6720k(1-c)+336kq_1\right)x^3-\frac{1680}{40320}kq_2(1-c)x^4\\
&-\frac{1}{240}q_2q_1x^5-\frac{1}{1440}kq_1q_2x^6+\frac{1}{5040}kq_1q_2x^7-\frac{1}{40320}kq_1q_2x^8
\end{aligned} \tag{24c}$$

where q_1 and q_2 are independent of variable x as:

$$q_1=-\frac{\alpha}{c^3(1+\delta)}-\frac{\beta\left(1-2\cosh(c\zeta_0)\frac{\phi_1}{\phi_2}+\left(\frac{\phi_1}{\phi_2}\right)^2\right)}{2\sinh^2(c\zeta_0)(1+\delta)} \tag{25a}$$

$$q_2=\frac{2\beta c^4\zeta_0\frac{\phi_1}{\phi_2}\cosh^2(c\zeta_0)-\beta c^4\zeta_0\left(1+\left(\frac{\phi_1}{\phi_2}\right)^2\right)\cosh(c\zeta_0)-\beta c^4\zeta_0\frac{\phi_1}{\phi_2}\sinh^2(c\zeta_0)-3\alpha\sinh^3(c\zeta_0)}{kc^4(1+\delta)\sinh^3(c\zeta_o)} \tag{25b}$$

The reminder of the solution, $R(x)$, can be evaluated by using Eq. (17). The reminder over the length of the beam ($R(x)$) and the maximum reminder (*MaxErr*) for a typical simply supported nanoactuator beam with α=1, β=1, ζ_0=1, ϕ_2/ϕ_1=0.1, δ=0.5 and k=30, adopted by Noghrehabadi et al. [8], is plotted in Figs. 3 and 4, respectively. Fig. 3 shows the distribution of $R(x)$ over the length of the beam, evaluated by use of two stages of ADM ($\phi_2(x)$), for the selected values of Duan's parameter. It is seen that the reminder $R(x)$ is comparatively uniform and small in the case of c=0.9. The variation of the maximum reminder for different stages of ADM and selected values of c is depicted in Fig. 4. There is an optimum value of c around 0.9 in the case of ϕ_2, but the optimum value of Duan's parameter (c) shifts to the values smoothly lower than 0.9 in the case of three and four stages of ADM (*i.e.*, ϕ_3 and ϕ_4). The vertical line in Fig. 4 indicates the constant line of c=1, which is the case of simple Duan-Rach ADM without using the Duan's parameter. Clearly, the optimum value of Duan's parameter, c≈0.9 is capable of declining the maximum reminder (error) of the two-stage solution (ϕ_2) to the values lower than that of the four-stage solution, evaluated without using Duan's parameter (*i.e.* ϕ_4 when c=1).

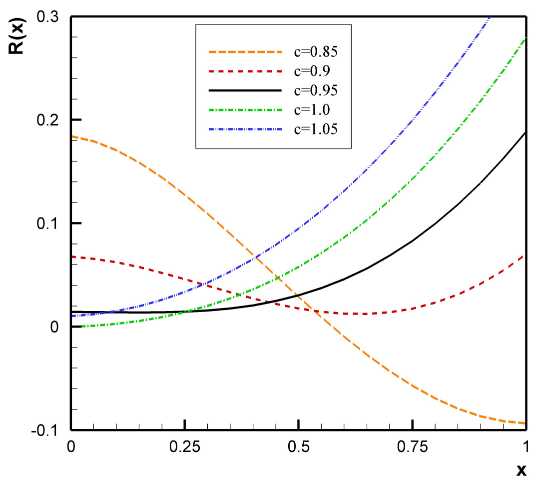

Fig. 3. Variation of the error reminder ($R(x)$) over the length of a SS nanoactuator for selected values of Duan's parameter for $\phi_2(x)$ when α=1, β=1, ζ_0=1, ϕ_2/ϕ_1=0.1, δ=0.5 and k=30.

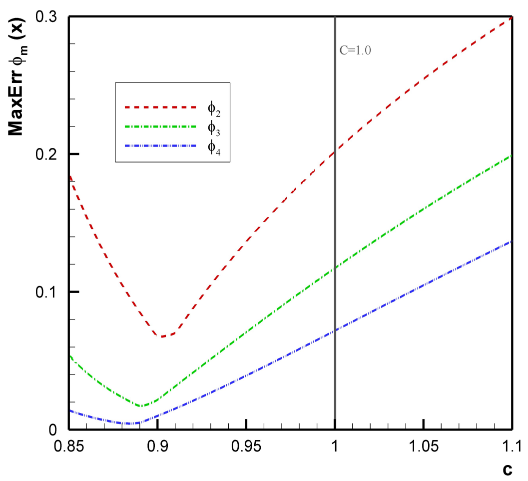

Fig. 4. Maximum value of reminder (MaxErr) of a *SS* nanoactuators as a function of Duan's parameter (c) for different stages of Adomian polynomials when α=1, β=1, ζ_0=1, ϕ_2/ϕ_1=0.1, δ=0.5 and k=30.

The tip deflection of the nanoactuator, evaluated by using different series sizes of Duan-Rach ADM and selected values of Duan's parameter, is shown in Table 2. A comparison between the results of the present study and those of FEM and Waswaz-ADM [8] is performed in Table 2, indicating that there is an excellent agreement between the two sets of results. Besides, a comparison between the evaluated tip deflection using FEM and our results shows that the increase of Adomian stages raises the accuracy of the solution. In agreement with Fig. 4, the tip deflections evaluated by using c=0.9 are remarkably accurate.

Table 2. Tip deflection of a SS cantilever nanoactuator when α=1,β=1, ζ_0=1, ϕ_2/ϕ_1=0.1, δ=0.5 and k=30.

c	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5
0.85	0.19167	0.17556	0.17235	0.17069	0.17018
0.9	0.16197	0.16527	0.16803	0.16910	0.16959
0.95	0.13000	0.15548	0.16273	0.16608	0.16779
1.0	0.11843	0.14594	0.15696	0.16235	0.16530
1.05	0.10229	0.13666	0.15096	0.15824	0.16239
MAD [8]	-	-	-	0.16763	0.16983
FEM [8]			0.17004		

3.3 Method of Undetermined Coefficients

In the method of undetermined coefficients, the inverse operator is defined in one initial point as $L^{-1}(.)=\int_0^x\int_0^x\int_0^x\int_0^x(.)\,dxdxdxdx$. Applying the inverse operator to the governing equation yields:

$$u(x)=u(0)+xu'(0)+\frac{x^2}{2}u''(0)+\frac{x^3}{6}u'''(0)+L^{-1}Nu \tag{26}$$

As the values of $u''(0)$ and $u'''(0)$ are unknown, the above equation is written as:

$$u(x)=u(0)+xu'(0)+\frac{x^2}{2}C_1+\frac{x^3}{6}C_2+L^{-1}Nu \tag{27}$$

where $u(0)=1$ and $u'(0)=0$, and C_1 and C_2 are the undetermined constant coefficients. The recursive scheme including undetermined parameters of C_1 and C_2 is written as [22]:

$$u_0(x)=1 \tag{28a}$$

$$u_1(x)=\frac{x^2}{2}C_1+\frac{x^3}{6}C_2+L^{-1}A_0 \tag{28b}$$

$$u_{m+1}(x)=L^{-1}A_m \tag{28c}$$

The values of C_1 and C_2 should be determined later by using the remaining boundary conditions, i.e., $u''(1)=0$ and $u'''(1)=0$. In the method of undetermined coefficients, evaluating C_1 and C_2 leads to a system of algebraic equations. For example, the case of the cantilever beam with α=1, β=1, ζ_0=1, ϕ_2/ϕ_1=0.1 and δ=0.5 and considering five stages of ADM (i.e. ϕ_5) lead to the following system of algebraic equations, which should be solved simultaneously for C_1 and C_2:

$$\begin{aligned}&-0.0078825C_1^4+(0.0189910-0.0085991C_2)C_1^3\\&+(0.0148533C_2-0.0427404-0.0035830C_2^2)C_1^2\\&+(1.107494590-0.0006737C_2^3-0.0204993C_2+0.0039787C_2^2)C_1\\&+0.0003630C_2^3+1.021671283C_2-0.4209466244-0.0000481C_2^4-0.0025773C_2^2=0\end{aligned} \tag{29a}$$

$$\begin{aligned}&-0.0788252C_1^4+(0.1586500-0.0945903C_2)C_1^3\\&+(0.1392384C_2-0.2673374-0.0429956C_2^2)C_1^2\\&+(0.443588-0.0087584C_2^3-0.1493122C_2+0.0413410C_2^2)C_1\\&+0.0041396C_2^3+1.1118036C_2-0.8540645-0.0006737C_2^4-0.0214097C_2^2=0\end{aligned} \tag{29b}$$

The coefficients of C_1 and C_2 are a function of non-dimensional parameters of α, β, ζ_0, ϕ_2/ϕ_1, δ. In addition, in the case of SS nanoactuator they are a function of the stiffness parameter k. Thus, Eqs. (29a) and (29b) should be solved for each combination of the non-dimensional parameters. Hence, obtaining an explicit relation for buckling of nanoactuators is not possible using the method of undetermined coefficients. Using the new Duan-Rach ADM to obtain an explicate relation is possible as there is not any undetermined coefficient in the obtained solution or the recursive scheme. The new Duan-Rach ADM method facilitates simple explicit relations between the behavior of the nanoactuator and non-dimensional parameters. For example, consider a cantilever nanoactuator with prescribed non-dimensional values of $\zeta_0=1$, $\phi_2/\phi_1=0.1$ and $\delta=0.5$. If an engineer needs to know the behavior of the tip deflection (u_{tip}) of the actuator as a simultaneous function of the non-dimensional van der Waals parameter (α) and the non-dimensional electrochemical parameter (β), using two stages of Duan-Rach ADM method (i.e., ϕ_2 and $c=0.9$), he/she can obtain such a relation as:

$$u_{tip}\left(\alpha,\beta\right)=1.0-0.028554\alpha^2+(-0.013738\beta-0.076208)\alpha+-0.00165\beta^2-0.019806\beta \tag{30}$$

or considering a more special case with $\alpha=1$ reduces Eq. (30) to:

$$u_{tip}=0.895238-0.033544\beta-0.001650\beta^2 \tag{31}$$

Eq. (30) shows the variation of u_{tip} as a function of the electrochemical parameter (β) for the mentioned nanoactuator when is α also prescribed. Such direct relations are crucial for design of nano sensors and sensing applications. For instance, consider a case in which the mentioned cantilever nanoactuator (i.e., $\alpha=1$, $\zeta_0=1$, $\phi_2/\phi_1=0.1$ and $\delta=0.5$) is utilized as a sensor for detecting the electrochemical potential of an electrolyte liquid (β). Assume that the tip deflection of the nanoactuator is found to be 0.8526 (i.e., y_{tip}= 0.8526) in an experiment after the nanoactuator is immersed in a liquid. In order to obtain the unknown value of the electrochemical parameter (β), Eq. (31) can be easily utilized. Substitution 0.8526 for u_{tip} in Eq. (31) and solving for β leads to β= -21.53 and β = 1.20. It is clear that negative values of β lack any physical meaning, and hence, the non-dimensional electrochemical parameter of the electrolyte is 1.2.

Considering five stages of Duan-Rach ADM method (i.e., ϕ_5 and $c=0.9$) for the mentioned nanoactuator (i.e. $\alpha=1$, $\zeta_0=1$, $\phi_2/\phi_1=0.1$ and $\delta=0.5$) simply results in:

$$\begin{aligned}u_{tip}\left(\beta\right)&=0.8971843-4.200829\times10^{(-7)}\beta^5-0.1349953\times10^{-4}\beta^4\\&-0.2093700\times10^{-3}\beta^3-0.2368345\times10^{-2}\beta^2-0.0329105\beta\end{aligned} \tag{32}$$

Nevertheless, the ADM method with undetermined coefficients, for the same nanoactuator, results in the following equation:

$$\begin{aligned}u_{tip}\left(\beta\right)&=-3.211405\times10^{-14}\beta^5+(1.539774\times10^{-11}C_1-1.007992\times10^{-11}+3.794155\times10^{-12}C_2)\beta^4\\&+\begin{pmatrix}-2.784431\times10^{-9}C_1^2+(-1.3528511\times10^{-9}C_2+4.1051879\times10^{-9})C_1\\+9.5525174\times10^{-10}C_2-1.6744630\times10^{-10}C_2^2-3.687925\times10^{-9}\end{pmatrix}\beta^3\\&+\begin{pmatrix}(2.266666\times10^{-7}C_1^3+(1.6186686\times10^{-7}C_2-5.7968076\times10^{-7})C_1^2\\+(3.9449818\times10^{-8}C_2^2-2.6126348\times10^{-7}C_2+0.12689219\times10^{-5})C_1\\+2.6192449\times10^{-7}C_2-3.0392996\times10^{-8}C_2^2+3.2710407\times10^{-9}C_2^3-0.21504976\times10^{-5}\end{pmatrix}\beta^2\\&+\begin{pmatrix}(-0.7106659\times10^{-5}C_1^4+(0.29529811\times10^{-4}-0.65599910\times10^{-5}C_2)C_1^3\\+(-0.12727611\times10^{-3}+0.18993713\times10^{-4}C_2-0.23428539\times10^{-5}C_2^2)C_1^2\\+(0.70711740\times10^{-3}-0.47585197\times10^{-4}C_2+0.42610086\times10^{-5}C_2^2-3.8179841\times10^{-7}C_2^3)C_1\\-2.3862401\times10^{-8}(C_2^2+23.747528C_2+492.97766)\times(C_2^2-37.598701C_2+601.02897)\end{pmatrix}\beta\\&-0.52609428\times10^{-4}C_1^4+(0.17181617\times10^{-3}-0.48562549\times10^{-4}C_2)C_1^3\\&+(-0.17343768\times10^{-4}C_2^2-0.60827510\times10^{-3}+0.10985922\times10^{-3}C_2)C_1^2\\&+(-0.22634039\times10^{-3}C_2-0.28263917\times10^{-5}C_2^3+0.24517296\times10^{-4}C_2^2+0.50280129)C_1\\&+0.97218888+0.18931067\times10^{-5}C_2^3+.016706835C_2-0.22731018\times10^{-4}C_2^2-1.7664948\times10^{-7}C_2^4\end{aligned} \tag{33}$$

It is worth noting that evaluating the chemical parameter (β) of the mentioned electrolyte liquid, using the ADM with undetermined coefficients, leads to a set of three nonlinear algebraic equations, one for β (Eq. 33) and two for the undetermined coefficients of C_1 and C_2 (which are similar to Eqs. (28a) and (28b) but include β as a variable in the coefficients). Solving such a nonlinear set of algebraic equations is tedious. However, using the new Duan-Rach ADM method solely Eq. (32) must be solved for β. It is clear that evaluating more complicated measurements with two unknown parameters (for example, ζ_0 and β) in a set of measurements (at least two different measurements) is more conveniently possible by using the Duan-Rach ADM method.

4. Conclusion

In this study, the Duan-Rach modified Adomian decomposition method was successfully applied to obtain an explicate solution for the buckling of nanoactuators in liquid electrolytes for two cases of cantilever beams and simply supported beams. The new modified Adomian decomposition method avoids undetermined coefficients and results in a fully explicate solution. The Duan's parameter is embedded into the ADM to accelerate the solution convergence. The results are compared with those obtained through the ADM of undetermined coefficients and the numerical solution of FEM available in the literature and excellent agreement was found between them. The error of the series solution is evaluated using its reminder. The results show that as the series size increases, the accuracy of the solution increases, as well. It was also found that using the proper value of Duan's parameter can significantly accelerate the convergence of the solution. The explicate solution obtained by the new ADM approach provides convenience analytical relations for the behavior of the nanoactuator as a function of different affective parameters.

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