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Short communication

Natural-convection heat and mass transfer from a vertical cone in porous media filled with nanofluids using the practical ranges of nanofluids thermo-physical properties

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ABSTRACT

The present study aims to analyze the effects of Brownian motion and thermophoresis forces on the natural convection heat transfer of nanofluids around a vertical cone placed in a saturated porous medium. The range of non-dimensional parameters and the definition of two important parameters of heat and mass transfer are discussed. The results show that the range of Lewis number as well as Brownian motion and thermophoresis parameters and also the definition of the reduced Nusselt and Sherwood numbers, used in the previous analyses, should be reconsidered. In the present study, reasonable definitions of reduced Nusselt and Sherwood numbers have been proposed and discussed in details. In contrast with previous researches, the present results show that the heat transfer associated with migration of nanoparticles is negligible compared with heat conduction and convection mechanisms.

Keywords: Natural convection; Porous media; Nanofluids; Brownian motion; Thermophoresis; Similarity solution

1. Introduction

Convective flow in porous media has a wide range of engineering applications, such as solar energy collectors, heat exchangers, geothermal and hydrocarbon recovery (Nield and Bejan, 2013; Vadasz, 2008; Gorla et al., 2011).

Nanofluids are produced by suspending nanoparticles, metallic or nonmetallic particles with the size of nanometers, in conventional heat transfer fluids such as water, oil, and ethylene glycol (Das et al., 2007). The thermal conductivity of conventional heat transfer fluids is low. However, presence of solid nanoparticles can enhance the thermal conductivity of base fluids (Ismay et al., 2013). The thermal conductivity and viscosity of seven nanofluids have been extensively examined by Buongiorno et al. (2009) and Venerus et al. (2010) in different laboratories. It is found that two of seven nanofluids showed shear-thinning behavior; the remaining five showed Newtonian behavior. Buongiorno (2006) has developed an analytical model for convective transport in nanofluids in which Brownian motion and thermophoresis effects are taken into account. Recently, Nield and Kuznetsov (2009) have used the Boungiorno's model (Buongiorno, 2006) to simulate the natural convection flow of nanofluids. Nield and Kuznetsov have examined the effect of nanoparticles on the natural convection boundary-layer flow past a vertical plate.

Very recently, Rashad et al. (2011) have investigated the natural convection boundary layer of a nanofluid about a permeable vertical full cone embedded in a saturated porous medium. Moreover, natural convective boundary layer flow over a horizontal plate embedded in porous media saturated with a nanofluid was investigated by Gorla and Chamkha (2011). In previous researches (Nield and Kuznetsov, 2009; Rashad et al., 2011; Gorla and Chamkha, 2011; Noghrehabadi et al., 2013a,b), the appropriate range of Brownian motion parameter, thermophoresis parameter, buoyancy ratio

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Nomenclature

D_B	Brownian diffusion coefficient (m^2/s)
D_T	thermophoretic diffusion coefficient (m^2/s)
f	rescaled nanoparticle volume fraction, nanoparticle concentration
g	gravitational acceleration vector (m/s^2)
κ	permeability of porous medium (m^2)
k	thermal conductivity (W/m K)
k_m	effective thermal conductivity of the porous medium (W/m K)
Le	Lewis number
Nb	Brownian motion parameter
Nr	buoyancy ratio
Nt	thermophoresis parameter
P	pressure (Pa)
Ra_x	local Rayleigh number
S	dimensionless stream function
T	temperature (K)
T_∞	ambient temperature (K)
T_W	wall temperature of the vertical cone (K)
U	reference velocity (m/s)
u, v	Darcy velocity components (m/s)
(x, y)	Cartesian coordinates (m)

Greek symbols

$(\rho c)_f$	heat capacity of the fluid (kg/m^3 K)
$(\rho c)_m$	effective heat capacity of porous medium (kg/m^3 K)
$(\rho c)_p$	effective heat capacity of nanoparticle material (kg/m^3 K)
μ	viscosity of fluid (Pa s)
α_m	thermal diffusivity of porous media (m^2/s)
β	volumetric expansion coefficient of fluid (1/K)
γ	cone half-angle (rad)
ε	porosity
η	dimensionless distance
θ	dimensionless temperature
ρ_f	fluid density (kg/m^3)
ρ_p	nanoparticle mass density (kg/m^3)
τ	parameter defined by Eq. (5)
φ	nanoparticle volume fraction
φ_∞	ambient nanoparticle volume fraction
φ_w	nanoparticle volume fraction at the cone surface
ψ	stream function

parameter and Lewis number have not been fully discussed. In addition, the effect of mass transfer on the reduced Nusselt number and the effect of temperature gradient on the reduced Sherwood number have not been taken into account.

In the present study, the influence of practical range of nanofluid parameters on the natural convection flow about a vertical cone placed in a saturated porous medium is analyzed. In addition, the practical range of non-dimensional nanofluid parameters and the reasonable definition of reduced Nusselt and Sherwood numbers are discussed.

2. Mathematical model

Consider a two-dimensional natural convection flow of a nanofluid about a vertical isothermal cone placed in a Darcy porous medium, in which the flow is incompressible and steady-state. It is assumed that the temperature and nanoparticle volume fraction (φ) at the cone surface ($y = 0$) have a fixed value of T_w and φ_w, respectively. The ambient values of T and φ, as y tends to infinity, are denoted by T_∞ and φ_∞, respectively.

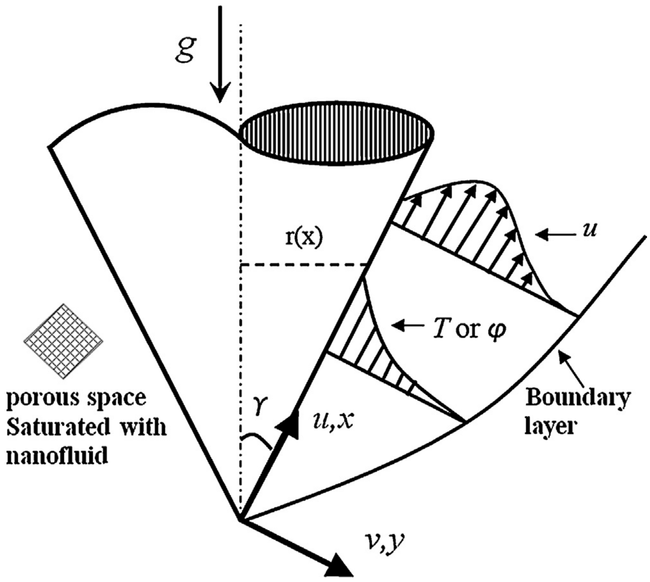

Fig. 1 – Schematic view and the coordinate system utilized to model the boundary layer configuration.

It has been demonstrated that the Brownian motion and thermophoresis forces induce slip velocities relative to the base fluid molecules (Buongiorno, 2006). Therefore, the concentration of nanoparticles in the convective heat transfer of nanofluids may not remain homogeneous. The thermophoresis force tends to move the particles in the direction opposite to the temperature gradient, and in contrast, the Brownian motion force tends to move the particles from high concentration to low concentration areas and homogeneous the fluid. As there is the temperature gradient in the boundary layer, the thermophoresis and Brownian motion forces appear, and hence, there is a boundary layer for concentration of nanoparticles.

The coordinate system is chosen such that x-axis is aligned with the flow on the cone surface. The physical model and coordinate system are shown in Fig. 1. There are three distinct boundary layers which are hydrodynamic boundary layer, thermal boundary layer and nanoparticle concentration boundary layer. It is worth noticing that the presence of hydrodynamic boundary layers is because of the thermal boundary layer. The thermal boundary layer induces the density difference in the fluid, and hence, the hydraulic boundary layer appears. Indeed, the presence of the hydraulic boundary layer is not because of the no-slip boundary condition or development of shear stresses due to the wall effects; it is because of the buoyancy forces. In addition, as the flow is buoyancy driven, and the practical temperature difference in the nanofluids is limited, the velocities are sufficiently low; hence the surface drag forces (Darcy term) are dominant, and they are in balance with the buoyancy forces. Therefore, in the present study, the flow in the porous medium with porosity ε and permeability κ is considered as Darcy flow, and the Oberbeck–Boussinesq approximation is applied.

Using the standard boundary layer approximations, the steady-state conservation of total mass, momentum and energy, as well as conservation of nanoparticles for nanofluids are presented respectively as follows:

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial y}=0, \tag{1}$$

$$\frac{\partial p}{\partial y}=0, \tag{2a}$$

$$\frac{\mu}{\kappa}u=\left[-\left(\frac{\partial p}{\partial x}-(1-\phi_\infty)\beta g\rho_{f_\infty}\cos\gamma(T-T_\infty)\right)\right.$$
$$\left.-(\rho_P-\rho_{f_\infty})g\cos\gamma(\phi-\phi_\infty)\right], \tag{2b}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial\phi}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{3}$$

$$\frac{1}{\varepsilon}\left[u\frac{\partial\phi}{\partial x}+v\frac{\partial\phi}{\partial y}\right]=D_B\frac{\partial^2\phi}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

α_m and τ, in the above equations, is defined as:

$$\alpha_m=\frac{k}{(\rho c)_f},\quad \tau=\frac{\varepsilon(\rho c)_p}{(\rho c)_f} \tag{5}$$

Based on the problem description, the boundary conditions are:

$$v=0,\quad T=T_w,\quad \phi=\phi_w,\quad \text{at}\quad y=0,\quad x\geq 0 \tag{6}$$

$$u=v=0,\quad T\to T_\infty,\quad \phi\to\phi_\infty,\quad \text{at}\quad y\to\infty \tag{7}$$

It is assumed that the thermal boundary layer is thin; hence, r can be approximated by the local radius of the cone ($r = x\sin\gamma$). Eqs. (2a) and (2b) are simplified using cross-differentiation, and the continuity equation will also be satisfied by introducing a stream function (ψ):

$$u=\frac{1}{r}\frac{\partial\psi}{\partial y},\quad v=-\frac{1}{r}\frac{\partial\psi}{\partial x}, \tag{8}$$

To simplify the system of equations, Eqs. (1)–(4) and attain a similarity solution for Eqs. (1)–(4) subject to Eqs. (6) and (7), the similarity variable (η) and the dimensionless similarity quantities S, f and θ are defined as,

$$\eta=\frac{y}{x}Ra_x^{1/2},\quad S=\frac{\psi}{\alpha_m\cdot r\cdot Ra_x^{1/2}},\quad f=\frac{\phi-\phi_\infty}{\phi_w-\phi_\infty},\quad \theta=\frac{T-T_\infty}{T_w-T_\infty} \tag{9}$$

and the local Rayleigh number (Ra_x) is as,

$$Ra_x=\frac{(1-\phi_\infty)\rho_{f\infty}\beta\kappa g x\cos\gamma(T_w-T_\infty)}{\mu_\infty\alpha_m} \tag{10}$$

By using the similarity variable, Eqs. (1)–(4) are reduced to the following three ordinary differential equations,

$$S''-\theta'+Nr\cdot f'=0 \tag{11}$$

$$\theta''+\frac{3}{2}\cdot S\theta'+Nb\cdot f'\cdot\theta'+Nt(\theta')^2=0 \tag{12}$$

$$f''+\frac{3}{2}\cdot Le\cdot S\cdot f'+\frac{Nt}{Nb}\theta''=0 \tag{13}$$

subject to the following boundary conditions,

$$\eta=0:\quad S=0,\quad \theta=1,\quad f=1 \tag{14a}$$

$$\eta\to\infty:\quad S'=0,\quad \theta=0,\quad f=0 \tag{14b}$$

where the non-dimensional parameters in the above equation are as follow:

$$Nr=\frac{(\rho_p-\rho_{f\infty})(\phi_w-\phi_\infty)}{(1-\phi_\infty)\rho_{f\infty}\beta(T_w-T_\infty)},\quad Nb=\frac{\varepsilon(\rho c)_p D_B(\phi_w-\phi_\infty)}{(\rho c)_f\alpha_m},$$
$$Nt=\frac{\varepsilon(\rho c)_p D_T(T_w-T_\infty)}{(\rho c)_f\alpha_m T_\infty},\quad Le=\frac{\alpha_m}{\varepsilon D_B}, \tag{15}$$

Here, Nr, Nb, Nt and Le denote the buoyancy ratio parameter, the Brownian motion parameter, the thermophoresis parameter and the Lewis number, respectively.

The quantities of local Nusselt number (Nu_x) and local Sherwood number (Sh_x) which interested in thermal engineering design of industrial equipments are defined by,

$$Nu_x=\frac{q_w x}{k(T_w-T_\infty)},\quad Sh_x=\frac{q_m x}{D_B(\phi_w-\phi_\infty)}, \tag{16}$$

where q_w is the wall heat flux, and q_m is the wall mass flux. Studying the exact concept of q_w shows that the heat from the surface transfers by not only conduction but also migration of nanoparticles. Similarly, the mass transfer is associated via both of the Brownian motion and thermophoresis forces. In the previous studies (Nield and Kuznetsov, 2009; Rashad et al., 2011; Gorla and Chamkha, 2011; Noghrehabadi et al., 2013a,b), the effect of nanoparticles migration on the heat transfer has been overlooked. Similarly the effect of thermophoresis on the mass transfer has not been considered. Therefore, the definitions of the reduced Nusselt and Sherwood numbers presented in previous works (Nield and Kuznetsov, 2009; Rashad et al., 2011; Gorla and Chamkha, 2011; Noghrehabadi et al., 2013a,b) should be reconsidered. In the work of Buongiorno (2006) which is the strong reference adopted by previous works (Nield and Kuznetsov, 2009; Rashad et al., 2011; Gorla and Chamkha, 2011) for analysis of heat and mass transfer associated by nanofluids, the quantities of q_w and q_m have been defined as follows:

$$q_w=-k\nabla T+\varepsilon h_p\cdot j_p \tag{17a}$$

$$q_m=\frac{j_p}{\rho_p} \tag{17b}$$

where q_w is the energy flux calculated as the sum of the conduction heat flux ($k\nabla T$) and the heat flux due to nanoparticle diffusion ($h_p\cdot j_p$). Here, q_m is the nanoparticle flux (mass flux), and $h_p = c_pT$ (Buongiorno, 2006). The quantity of j_p is the diffusion mass flux for the nanoparticle. j_P is the sum of two important diffusion terms (i.e. Brownian diffusion and thermophoresis) which is written as (Buongiorno, 2006),

$$j_P=-\rho_p D_B\nabla\phi-\rho_p D_T\frac{\nabla T}{T} \tag{18}$$

Table 1 – The range of nanofluid parameters, i.e. *Nb*, *Nr*, *Nt* and *Le*.

Parameters	Range of variation
Nb	2.4E−9 to 1E−4
Nr	0 to +9.7E+2
Nt	4E−9 to 1.3E−4
Le	1E+3 to 3E+6

Using similarity transforms, Eq. (9), and substituting the Eqs. (17a), (17b) and (18) in the Eq. (16), the reduced Nusselt number (Nu_r) and reduced Sherwood number (Sh_r) are obtained as,

$$Nu_r = Nu_x Ra_x^{-1/2} = -(1+Nt)\theta'(0) - Nb \cdot f'(0), \tag{19a}$$

$$Sh_r = Sh_x Ra_x^{-1/2} = -f'(0) - \frac{Nt}{Nb}\theta'(0). \tag{19b}$$

2.1. *Physical range of parameters*

In the engineering applications, it is required to make sufficiently accurate estimations of heat transfer rates. Reviewing the previous studies related to the natural convection of nanofluids in the literature reveals that the adopted range of nanofluid parameters is not adequate. In order to perform a realistic analysis on the heat transfer of nanofluids, the practical range of non-dimensional parameters of Brownian motion (Nb), thermophoresis (Nt), buoyancy ratio (Nr) and Lewis number (Le) for nanofluids should be demonstrated. To aim this purpose, first, the practical range of thermo-physical properties (i.e. D_B, D_T, $(\rho c)_p/(\rho c)_f$, β, α_m, etc.) which affects the non-dimensional parameters should be analyzed.

For water base nanofluids at room temperature with nanoparticles of 100 nm diameters, the Brownian diffusion coefficient, D_B, ranges from 1×10^{-10} to 1×10^{-12} m^2/s (Buongiorno, 2006). The thermophoresis coefficient, D_T, also ranges from 1×10^{-10} to 1×10^{-12} (Buongiorno, 2006; Hwang et al., 2009). It is assumed that the temperature difference varies between 5 and 40 °C, and the nanoparticle volume fraction difference varies between 0 to 0.1. The porosity is in the range of 0–1. For regular solid and liquid materials β is in the order of 10^{-5} to 10^{-3} and $(\rho c)_p/(\rho c)_f$ is in the order of 0.1. The effective thermal diffusivity of saturated porous media ranges from 10^{-7} to 10^{-8}. For instance, the volumetric thermal expansion of Al_2O_3-water nanofluid at room temperature with 4% volume fraction of nanoparticles is 2×10^{-3} and the $(\rho c)_p/(\rho c)_f$ is calculated as 0.0640; the thermal diffusivity of Al_2O_3–water is evaluated as 1.5991×10^{-7} (40 nm particles) and 1.4560×10^{-3} (100 nm particles) (Khanafer and Vafai, 2011). It is worth noticing that the volumetric thermal expansion and the thermal capacity of nanofluids are almost independent of the size of nanoparticles. By considering these ranges of the thermo-physical properties, the approximate ranges of nanofluid parameters are obtained and summarized in Table 1. In the present study, we are interested in the case in which heat transfer is driving the flow rather than the mass transfer. Therefore, the values of buoyancy-ratio parameter Nr are assumed very smaller than unity.

In the literature (Nield and Kuznetsov, 2009; Rashad et al., 2011; Gorla and Chamkha, 2011; Noghrehabadi et al., 2013a,b), for example, in the work of Nield and Kuznetsov (2009) and Gorla and Chamkha (2011), the range of 0.1–0.5 has been chosen for nanofluid parameters, i.e. Nb, Nt and Nr to calculate the adopted model of their problem. In addition, they (Nield and Kuznetsov, 2009; Gorla and Chamkha, 2011) have considered the range of 1–1000 for Lewis number. Rashad et al. (2011) have adopted the range of 0.1–0.7 for Nb, Nt, Nr, and the range of 1–100 for Le. However, the results of Table 1 reveals that the range of nanofluid parameters in the previous works (Nield and Kuznetsov, 2009; Rashad et al., 2011; Gorla and Chamkha, 2011; Noghrehabadi et al., 2013a,b) is not in agreement with the practical range of conventional nanofluids.

Table 2 – Comparison of results for reduced Nusselt number $-\theta'(0)$.

Cheng et al. (1985)	Yih (1997)	Present results ($Nb = Nt = Nr = 0$)
0.7685	0.7686	0.76859

3. Results and discussion

The system of Eqs. (11)–(13) subject to the boundary conditions (14a) and (14b) is considered and numerically solved for various values of Nr, Nb, Nt and Le using fourth-order Runge–Kutta method associated with Gauss-Newton method. A maximum relative error of 10^{-5} is used as the stopping criteria for the iterations.

Neglecting the effects of thermophoresis, Brownian motion and buoyancy ratio parameters ($Nt = Nb = Nr = 0$), the present study reduces to the case of a pure fluid which was studied by Cheng et al. (1985) and Yih (1997). Table 2 shows a comparison between $-\theta'(0)$ obtained in the present work and the results reported by Cheng et al. (1985) and Yih (1997). The results of Table 2 show excellent agreement between the results of present study and previous studies.

Utilizing the practical range of nanofluid parameters, the results reveal that increasing the value of thermophoresis or Brownian motion parameter does not influence the velocity, temperature and nanoparticles concentration profiles as well as hydrodynamics, thermal and nanoparticle concentration boundary layers thickness. Moreover, simulation results show that the thickness of concentration boundary layer is very small in comparison with hydrodynamic and thermal boundary layers. The results demonstrate that variation of Lewis number would not significantly affect the velocity and temperature profiles, whereas it would decrease the concentration profiles.

The variation of the dimensionless heat transfer rate (i.e. reduced Nusselt number) with respect to the Brownian motion parameter (or thermophoresis parameter) for different values of the buoyancy ratio parameter (or Lewis number) is depicted in Fig. 2 (or Fig. 3). These figures illustrate that an increase of thermophoresis, Brownian motion, buoyancy ratio or Lewis number does not have a significant effect on the reduced Nusselt number. A reason for this behavior is that the coefficients of Nb and Nt appearing in Eqs. (12) and (19a) are very small. Therefore, the variation of these parameters within their practical range does not have a significant effect on the heat transfer rate. The variation of Lewis number affects the concentration profiles; however, again because of the very small value of Nb, the effect of concentration gradient on the reduced Nusselt number is diminished.

Fig. 4 (or Fig. 5) is plotted to show the variation of the reduced Sherwood number with respect to the Brownian motion parameter (or thermophoresis parameter) for different values of the buoyancy ratio parameter (or Lewis number). These figures illustrate that increase of thermophoresis

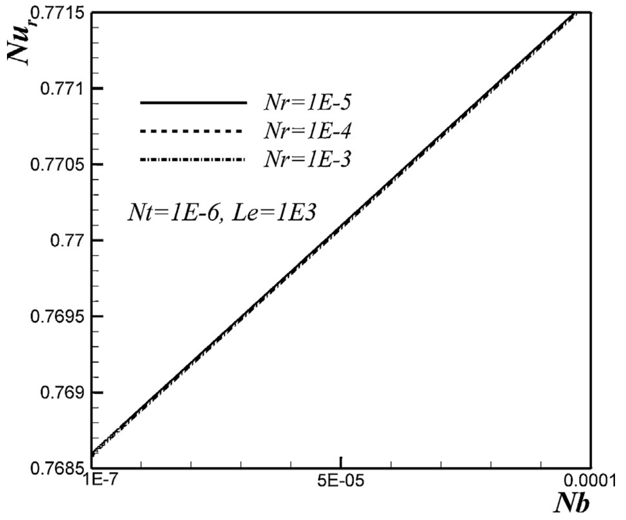

Fig. 2 – Reduced Nusselt number profiles for various values of buoyancy ratio and Brownian motion parameters.

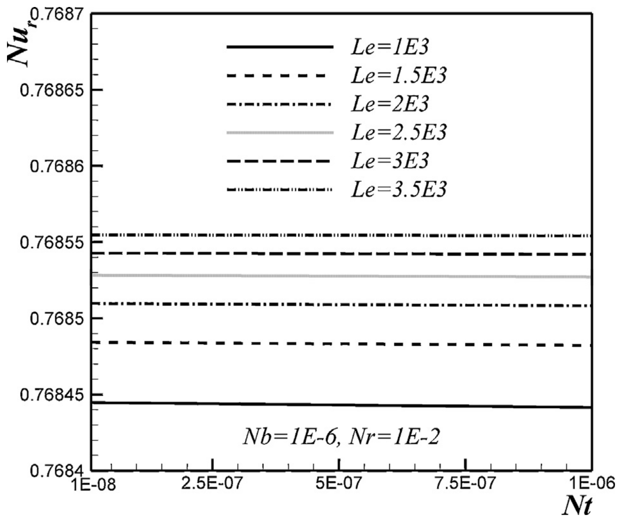

Fig. 3 – Reduced Nusselt number profiles for various values of Lewis number and thermophoresis parameters.

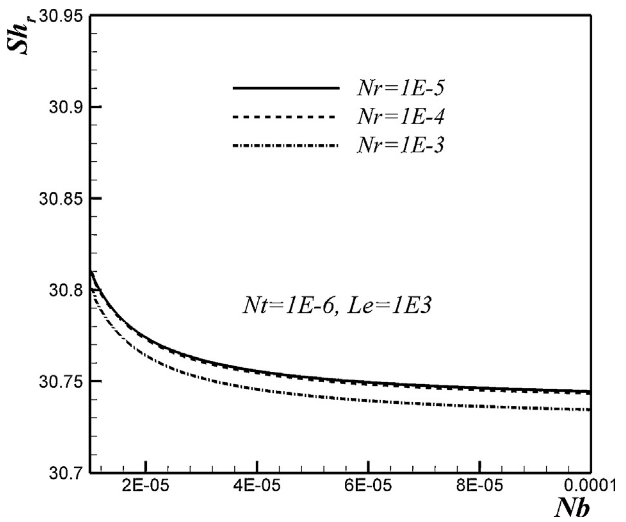

Fig. 4 – Reduced Sherwood number profiles for various values of buoyancy ratio and Brownian motion parameters.

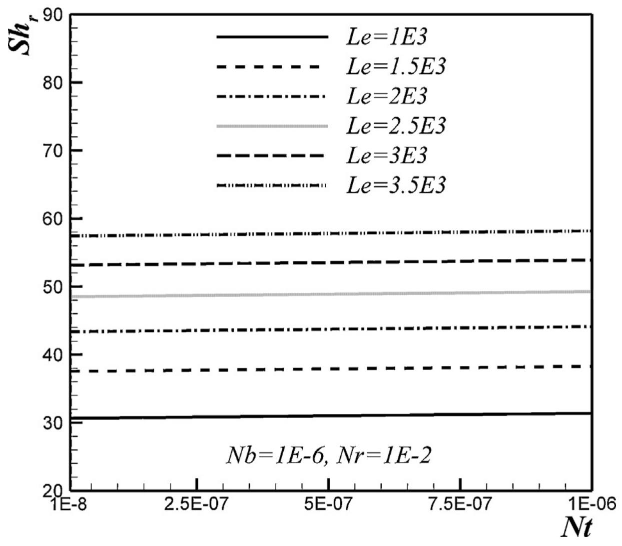

Fig. 5 – Reduced Sherwood number profiles for various values of Lewis number and thermophoresis parameter.

or Brownian motion parameters does not influence the magnitude of reduced Sherwood number. In addition, the results depict that increase of Lewis number (or buoyancy parameter) increases (or decreases) the magnitude of the reduced Sherwood number.

In the case of $Le = 1000$, the reduced Nusselt number, Nur, is computed for 8000 different cases. For the computations, Nb and Nt are adopted in the range of 10^{-7} to 10^{-4}, and Nr is adopted in the range of 10^{-3} to 10^{-1}. A linear regression was performed on the computed results. The linear relation is as follow:

$$Nur_{est} = 0.7686 + 0.051543\,Nb + 0.045099\,Nt - 0.19747\,Nr \quad (20)$$

where Nur_{est} shows the estimated value of reduced Nusselt number. The maximum relative error of this equation is 2.7%, which was calculated using $(Nu_{est} - Nur)/Nur$.

As mentioned, Nield and Kuznetsov (2009) have examined the natural convection heat transfer of nanofluids over a flat plate embedded in a porous medium saturated with a nanofluid. When $Le = 1000$, the linear regression reported by Nield and Kuznetsov (2009) can be written as follow:

$$Nur_{est} = 0.444 - 0.036\,Nb - 0.313\,Nt - 0.148\,Nr \quad (21)$$

Eq. (21) is valid when Nb, Nt and Nr in the range of 0.1–0.5. As seen, all of the coefficients of Nb, Nt and Nr are negative in Eq. (21). The negative values of the coefficients show that an increase of Nt, Nb and Nr would decrease the reduced Nusselt number. However, in the Eq. (20), the coefficient of Nb and Nt are positive. Hence, an increase of Nb and Nt would increase the reduced Nusselt number. Moreover, as the values of Nb and Nt are very small, the increase of Nusselt number is comparatively negligible. The main reason for the observed difference between Eqs. (20) and (21) is the contribution of migration of nanoparticles in the definition of reduced Nusselt number.

An important finding of the present study is that the present results do not confirm the results reported by previous studies; for example references (Gorla et al., 2011; Rashad et al., 2011; Gorla and Chamkha, 2011). The previous studies (Gorla et al., 2011; Rashad et al., 2011; Gorla and Chamkha, 2011) have reported a significant decreasing trend for the reduced Nusselt number by increase of thermophoresis, Brownian motion and buoyancy ratio parameters. The observed difference between the results of present study and the previous works is because of the fact that the effect of migration of nanoparticles from the surface has changed the trend of variation of reduced Nusselt number. In addition, in the work of mentioned studies (Gorla et al., 2011; Rashad et al., 2011; Gorla and Chamkha, 2011), a different physical range of parameters was adopted.

4. Conclusion

A combination of similarity and numerical approach is adopted to theoretically analyze the effect of nanoparticles on the natural convection heat transfer of nanofluids about a vertical cone embedded in a porous medium. The results can be summarized as follows:

I The physical range of nanofluid parameters, which has been used by many of previous researchers, is not in good agreement with the practical range of these parameters.

II Using practical range of non-dimensional parameters, the results demonstrate that the heat transfer associated with migration of nanoparticles is negligible in comparison with heat conduction and convection mechanisms.

III Reviewing the equations of the reduced Nusselt and Sherwood numbers presented by previous researchers in the literature and considering the exact concept of quantities of q_w and q_m show that the effect of heat transfer associated via migration of nanoparticles should be added to the definition of heat flux (q_w). Similarly, the effect of mass transfer because of temperature gradient should be added to the definition of mass flux (q_m).

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References

Buongiorno, J., 2006. Convective transport in nanofluids. J. Heat Transfer 128, 240–250.

Buongiorno, J., Venerus, D.C., Prabhat, N., McKrell, T., Townsend, J., et al., 2009. A benchmark study on the thermal conductivity of nanofluids. J. Appl. Phys. 106, 094312.

Cheng, P., Le, T.T., Pop, I., 1985. Natural convection of a darcian fluid about a cone. Int. Commun. Heat Mass Transfer 12, 705–717.

Das, S.K., Choi, S.U.S., Yu, W., Pradeep, T., 2007. Nanofluids: Science and Technology. John Wiley & Sons, Hoboken.

Gorla, R.S.R., Chamkha, A.J., 2011. Natural convective boundary layer flow over a horizontal plate embedded in a porous medium saturated with a nanofluid. J. Mod. Phys. 2, 62–71.

Gorla, R.S.R., Chamkha, A.J., Rashad, A.M., 2011. Mixed convective boundary layer flow over a vertical wedge embedded in a porous medium saturated with a nanofluid: Natural Convection Dominated Regime. Nanoscale Res. Lett. 207, 1–9.

Hwang, K.S., Jang, S.P., Choi, S.U.S., 2009. Flow and convective heat transfer characteristics of water-based Al_2O_3 nanofluids in fully developed laminar flow regime. Int. J. Heat Mass Transfer 52, 193–199.

Ismay, M.J.L., Doroodchi, E., Moghtaderi, B., 2013. Effects of colloidal properties on sensible heat transfer in water-based titania nanofluids. Chem. Eng. Res. Des. 91, 426–436.

Khanafer, K., Vafai, K., 2011. A critical synthesis of thermophysical characteristics of nanofluids. Int. J. Heat Mass Transfer 54, 4410–4428.

Nield, D.A., Bejan, A., 2013. Convection in Porous Media, fourth ed. Springer, New York.

Nield, D.A., Kuznetsov, A.V., 2009. The Cheng–Minkowycz problem for natural convective boundary-layer flow in a porous medium saturated by a nanofluid. Int. J. Heat Mass Transfer 52, 5792–5795.

Noghrehabadi, A., Behseresht, A., Ghalambaz, M., Behseresht, J., 2013a. Natural convection flow of nanofluids over a vertical cone embedded in a non-Darcy porous medium. J. Thermophys. Heat Transfer 27, 334–341.

Noghrehabadi, A., Behseresht, A., Ghalambaz, M., 2013b. Natural convection of nanofluid over vertical plate embedded in porous medium: prescribed surface heat flux. Appl. Math. Mech. 34, 669–686.

Rashad, A.M., El-Hakiem, M.A., Abdou, M.M.M., 2011. Natural convection boundary layer of a non-Newtonian fluid about a permeable vertical cone embedded in a porous medium saturated with a nanofluid. Comput. Math. Appl. 62, 3140–3151.

Vadasz, P., 2008. Emerging Topics in Heat and Mass Transfer in Porous Media. Springer, New York.

Venerus, D., Buongiorno, J., Christianson, R., Townsend, J., Bang, I.C., Chen, G., Zhou, S., 2010. Viscosity measurements on colloidal dispersions (nanofluids) for heat transfer applications. Appl. Rheol. 20, 445821–445827.

Yih, K.A., 1997. The effect of uniform lateral mass flux on free convection about a vertical cone embedded in a saturated porous media. Int. Commun. Heat Mass Transfer 24, 1195–1205.