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Mathematical study of Electroosmotically driven peristaltic flow of Casson fluid inside a tube having systematically contracting and relaxing sinusoidal heated walls

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A R T I C L E I N F O

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A B S T R A C T

The electro-osmotically driven flow of Casson fluid across a cylindrical geometry with systematically contracting and relaxing sinusoidal walls is mathematically interpreted. The phenomenon of heat transfer also has its worth in the present investigation. The Joule heating and viscous effects are also studied in detail. The mathematically formulated equations that govern the flow of Casson fluid are simplified in their dimensionless form by using lubrication approximation, i.e. ($\lambda \to \infty$). Finally, exact mathematical solutions are flourished for these dimensionless equations. The graphical results are presented to further explain the exact solutions of velocity, temperature, pressure rise, pressure gradient, and streamlines pattern. The major outcomes show that electro-osmosis is very helpful in controlling both flow and heat transfer.

1. Introduction

Most of the biological fluids are driven within tubes having flexible walls due to an organized compression and expansion of these flexible walls. This systematic compression and expansion produce a moving sinusoidal wave at the walls of such cylindrical tubes. This phenomenon of fluid transport is called Peristalsis. Barton and Raynor [1] had provided a mathematical investigation that explains the peristaltic flow across cylindrical tubes. In their work, firstly the wavelength of wall disruption is considered much greater than the tube's radius. Further, they had also considered the case study when both wavelength and radius are equally small. In the human body, the peristaltic mechanism plays an important role in the gastrointestinal region, ureter, bile carrying tube, and distinct glandular tubes as well. The roller pumps also function on the peristalsis principle and these are utilized in many ways to transport fluids like harmful fluids (Acids and bases etc), slurry (a mixture of fluid with distinct materials) and blood, etc [2]. The peristaltic flow with creeping flow approximation for their flow problem with a detailed study of disturbance amplitude effect, pressure gradient effects, and the streamlines pattern was provided by Pozrikidis [3]. The topic of peristalsis has huge importance due to its wide

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Nomenclature

$(\overline{R}, \overline{Z})$ (m)	Cylindrical coordinate system
$(\overline{U}, \overline{W})$ (m/s)	Velocities along radial and axial coordinate
a (m)	Radius of tube
b (m)	amplitude of sinusoidal wave
c (m/s)	Wave characteristic Speed
Φ	Amplitude to mean radius ratio
e (C)	Electronic charge
K_B (J/K)	Boltzmann Constant
λ_d (m)	Debye-Length
i_e (A/m^2)	Current density
n^+, n^-	cation and anion densities
$\overline{\Phi}$ (V)	Electro-kinetic potential function
k $\left(\frac{W}{mK}\right)$	Fluid thermal conductivity
U_{HS} (m/s)	Helmholtz Smoluchowski velocity
p_y (N/m^2)	Yield stress
EDL	Electric double layer
λ (m)	Wavelength
B_r	Brinkman number
z^* (C)	Charge balance
m	Electro-osmotic parameter in non-dimensional form
S	Joule heating in non-dimensional form
σ (Ωm)	Fluid electrical resistivity
$\rho_e (C/m)$	Ionic charge density
n_0	Ions concentration
$\in$ (F/m)	Permittivity
E_z (V/m)	Axial Electrical field
T^* (K)	Electrolyte solution average temperature
ζ (m^2/VS)	zeta potential
μ_B (cP)	Plastic viscosity
β	Casson parameter
t	Dimensionless time
p	Dimensionless pressure

applicability and many researchers have contributed in this area by investigating peristaltic flow problems for both Newtonian and non-Newtonian fluids as well. Bohme and Friedrich [4] had examined the viscoelastic fluid flow problem within a channel with a sinusoidal wave at walls moving along the channel's length. The axial symmetry, non-Newtonian liquid flow across a tube with a harmonic wave moving sinusoidally at the walls of the tube was interpreted by Siddiqui and Schwarz [5]. Further, many other researchers have contributed to the study of non-Newtonian liquid flow problems across tubes with peristaltic walls [6-8].

Electro-osmosis includes the study of the flow of liquids under externally applied electric field effects. It has a wide range of applications in fluid dynamics research. It is applicable in blood flow problems, medication, treatment of diseases like abnormalities in cells and sickle cells, etc [9]. The pumps that work on the electro-osmotic principle can easily generate liquid flow with high pressure and these pumps are cheap, efficient in working, and lightweight [10]. Lodhi and Ramesh [11] had provided a mathematical investigation that deals with the comparative study of electro-osmotic effects and MHD. Yang et al. [12] had explained the flow of liquids through channels having small lengths with electro-osmotic effects. The potential distribution of the applied electric field is considered by utilizing the Poisson-Boltzmann equations in their work. Santiago [13] had investigated the mathematical study of the electro-osmotic flow of liquids with constant properties of the fluid. The combined mathematical study of electro-osmosis and peristalsis has its worth due to engineering and medical applicability. Bandopadhyay [14] et al. had interpreted a mathematical investigation that describes the electro-kinetically modeled flow of liquids within a channel having peristaltic walls. Moreover, the electro-kinetically driven flow of distinct non-Newtonian liquids inside channels with sinusoidally advancing walls are recently studied by many researchers due to its applications. Further, the recent researches that convey the combine mathematical analysis of peristaltic flow and electro-osmosis are given [15-18]. Prakash and Tripathi [19] had examined the peristaltic flow of Williamson fluid across a small length channel with electro-osmotic effects. The mathematical analysis of bio rheological fluid flow inside a channel with wavy walls and electro-kinetic effects was presented by Tripathi [20] et al. Chaube [21] et al. had theoretically studied the electro-kinetically modeled flow of micropolar fluid across a channel with fluctuating walls. The electro-osmotic flow of Bingham fluid with joule heating effects inside a geometry having sinusoidally advancing walls was explained by Nadeem [22] et al. Some recent

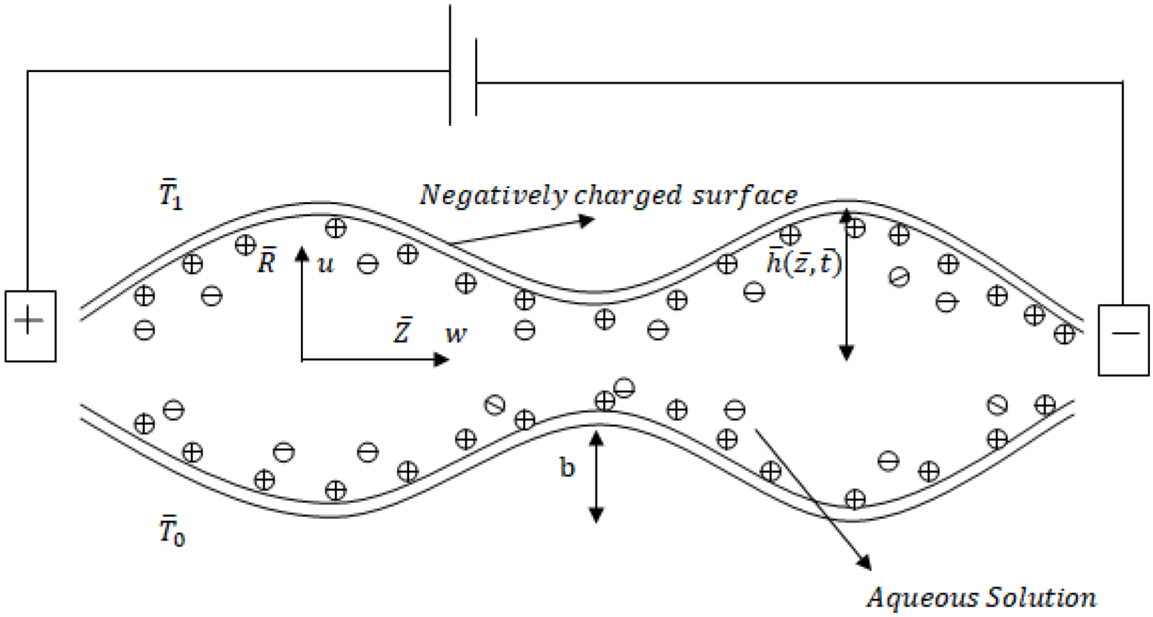

Fig. 1. Geometry of the formulated problem.

research studies that deal with the non-Newtonian Casson fluid flow in a tube with sinusoidal walls are provided [23-27]. We have investigated completely the already done research work for the combined study of peristalsis and electro-osmosis and it is observed that the non-Newtonian flow of Casson fluid inside a cylindrical geometry with moving walls and electro-osmotic effects is not explored mathematically.

This research article addresses the electro-osmotically driven flow of Casson fluid inside a cylindrical geometry with systematically contracting and relaxing sinusoidal walls for the very first time. The heat transfer analysis also has its worth in the present investigation. The joule heating and viscous effects are also studied in detail. The potential distribution of the applied electric field is considered by utilizing the Poisson-Boltzmann equation. The mathematically formulated equations that govern the flow of Casson fluid are simplified in their dimensionless form by using lubrication approximation, i.e. ($\lambda \to \infty$). Finally, exact mathematical solutions are flourished for these dimensionless equations. The graphical results are provided to further explain the exact solutions of velocity, temperature, pressure gradient, pressure rise, and streamlines pattern. Figure 1

2. Mathematical formulation

The electro-osmotically modeled flow of Casson fluid inside a cylindrical geometry with systematically contracting and relaxing sinusoidal walls is mathematically interpreted.

The geometrical representation of sinusoidally moving walls in mathematical form is provided as

$$\bar{h}(\bar{z},\bar{t}) = a + bSin\left(\frac{2\pi}{\lambda}\left(\bar{Z} - c\bar{t}\right)\right), \tag{1}$$

The governing mathematical equations [28] having dimensional form in a fixed frame are given

$$\frac{\partial \bar{U}}{\partial \bar{R}} + \frac{\bar{U}}{\bar{R}} + \frac{\partial \bar{W}}{\partial \bar{Z}} = 0, \tag{2}$$

$$\rho\left(\frac{\partial \bar{U}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{R}} + \bar{W}\frac{\partial \bar{U}}{\partial \bar{Z}}\right) = -\frac{\partial \bar{P}}{\partial \bar{R}} + \frac{1}{\bar{R}}\frac{\partial}{\partial \bar{R}}\left(\bar{R}\bar{S}_{\bar{R}\bar{R}}\right) + \frac{\partial}{\partial \bar{Z}}\left(\bar{S}_{\bar{R}\bar{Z}}\right), \tag{3}$$

$$\rho\left(\frac{\partial \bar{W}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{W}}{\partial \bar{R}} + \bar{W}\frac{\partial \bar{W}}{\partial \bar{Z}}\right) = -\frac{\partial \bar{P}}{\partial \bar{Z}} + \frac{1}{\bar{R}}\frac{\partial}{\partial \bar{R}}\left(\bar{R}\bar{S}_{\bar{R}\bar{Z}}\right) + \frac{\partial}{\partial \bar{Z}}\left(\bar{S}_{\bar{Z}\bar{Z}}\right) + \rho_e E_z, \tag{4}$$

$$\rho C_p\left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{T}}{\partial \bar{R}} + \bar{W}\frac{\partial \bar{T}}{\partial \bar{Z}}\right) = \bar{S}_{\bar{R}\bar{R}}\frac{\partial \bar{U}}{\partial \bar{R}} + \bar{S}_{\bar{R}\bar{Z}}\frac{\partial \bar{W}}{\partial \bar{R}} + \bar{S}_{\bar{Z}\bar{R}}\frac{\partial \bar{U}}{\partial \bar{Z}} + \bar{S}_{\bar{Z}\bar{Z}}\frac{\partial \bar{W}}{\partial \bar{Z}} + k\left(\frac{\partial^2 \bar{T}}{\partial \bar{R}^2} + \frac{1}{\bar{R}}\frac{\partial \bar{T}}{\partial \bar{R}} + \frac{\partial^2 \bar{T}}{\partial \bar{Z}^2}\right) + s, \tag{5}$$

Here energy equation contains s, (i.e. Joule heating effect in the dimensional form). Mathematically it is provided by $s = i_e^2\sigma$.