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Physical aspects of peristaltic flow of hybrid nano fluid inside a curved tube having ciliated wall

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ABSTRACT

The intensions of this work are to explain the peristaltic flow and heat transfer of hybrid (Cu-Ag/blood) and phase flow (Cu/blood) nanofluid models inside a curved tube having ciliated wall. The governing mathematical equations are established for a curvilinear coordinate system $(\overline{R},\overline{Z})$. In order to simplify these equations, we have exploited the lubrication approximation, i.e. Reynolds number tends to zero and wavelength tends to infinity. Finally, these abridged governing equations, subject to corresponding boundary conditions, are solved and luckily found exact solutions for the complicated problem. These solutions are discussed graphically for both hybrid (Cu-Ag/blood) and phase flow (Cu/blood) nanofluid models against various physical parameters that are involved in the governing equations. It is observed that the axial and thermal symmetry is demolished due to curved nature of the tube but when parameter s increases, i.e. the curved geometry is transformed into a straighter channel then the symmetry about the centre of channel is restored.

Introduction

Peristalsis is the mechanism of fluid transport inside a channel due to its sinusoidally advancing walls. The flow of fluid due to peristaltic pumping along the length of tube was discussed by Jaffrin [1]. Pozrikidis [2] had studied the two dimensional peristaltic flow for low value of Reynolds number and comparatively large value of wavelength. The trapping phenomena and reflux of peristaltic flow was discussed in detail by Takabatake et al. [3]. They had also described the physiological and engineering applications of peristaltic flows. The peristaltic flow in curved geometries, for different fluid models, was interpreted by Nadeem et al. [4-6].

There role of cilia is very essential in processes such as circulation, respiration, locomotion and food movement in digestive tract [7] and almost all the animal kingdoms have cilia. Blake [8] presented the study of cilia developed flows in a tube. These cilia co-ordinate in such a way that their sequential actions result in the formation of a metachronal wave that propagates the flow. Nadeem et al. [9] had modelled the problem for the peristaltic flow inside a two dimensional curved tube having ciliated walls. Ellahi et al. [10] studied the fluid flow through an annulus having blood clot and ciliated walls. The effect of sinusoidal wall motion on MHD flow of solid particles in a planar channel was described by Bhatti et al. [11]. Ramesh and Tripathi had done mathematical computations for cilia developed flow [12]. The metachronal wave developed due to cilia induced micro-polar fluid flow was studied by Farooq et al. [13].

Nanoparticles are tiny particles having size less than one hundred nanometre. In many industrial problems, the heat transfer analysis is limited due to low thermal conductivity of fluid but this issue can be resolved by enhancing the nanoparticles concentration in the fluid, since it finally increases the thermal conductivity of fluid [14]. The heat transfer for a two phase nanofluid flow inside a curved geometry was interpreted by Shahzadi et al. [15]. The ferro-nanofluid flow with torque effects between rotating circular plates is interpreted by Zhang et al. [16]. The flow inside a curved channel with nanofluid was mathematically

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Nomenclature

$R^*(m)$	Radius of curved channel
$\left(\overline{R},\overline{Z}\right)$	Curvilinear coordinates
$a\ (m)$	Mean radius of channel
$b(m)$	Amplitude of wave
$\lambda(m)$	Wavelength of metachronal wave
$c(ms^{-1})$	Wave speed
$\overline{Z}_o$	reference position of particle
α	Eccentricity of Elliptical Motion
hnf	Hybrid nanofluid
$\mu(Nsm^{-2})$	Dynamic viscosity
$k\left(\frac{W}{mK}\right)$	Thermal conductivity
$C_p\left(\frac{J}{kg.K}\right)$	Specific heat
$\rho(\frac{kg}{m^3})$	Density
$\alpha^*(K^{-1})$	Thermal expansion coefficient
$Q_o\left(\frac{W}{m^2K}\right)$	Constant of heat absorption parameter
β	Wave number
s	curvature parameter
γ	Heat generation/absorption coefficient
ϕ	Volume fraction of nanofluid 1.3%
G_r	Grashoff number
ϕ_1	Volume fraction of Copper (s_1)1%
ϕ_2	Volume fraction of Silver (s_2)0.3%
bf	Base fluid
R_e	Reynold number

modelled by Tripathi et al. [17]. The blood flow inside a micro-channel with different shapes of nano-particles was studied by Tripathi and Prakash [18]. Some other studies dealing the studies of nanofluids and heat transfer are discussed in the Refs. [5,19-23]. Further, some recent researches that use the same technique are given [24-31].

Hybrid nanofluid is a new class of nanofluids that is obtained by combining two or more nanomaterials in the base fluid [32]. In this work, we have studied the peristaltic flow of hybrid (Cu-Ag/blood) and phase flow (Cu/blood) nanofluid models inside a curved tube having ciliated wall. We have considered blood as base fluid. The governing mathematical equations are developed for a two-dimensional curvilinear coordinate system $(\overline{R},\overline{Z})$. In order to simplify these equations, we have applied the approximations $R_e\to 0, \lambda\to\infty$. Finally, these simplified governing equations, subject to corresponding boundary conditions, are solved to obtain exact solution for velocity and temperature profile by using Mathematica software. These solutions are discussed graphically for both hybrid (Cu-Ag/blood) and phase flow (Cu/blood) nanofluid models against various physical parameters that are involved in the governing equations. Streamlines are also plotted.

Mathematical formulation

Considered the peristaltic flow of an incompressible, laminar, Newtonian fluid inside a curved channel, with centre O and radius R^*, having ciliated walls. The governing equations are formulated for a curvilinear coordinate system $(\overline{R},\overline{Z})$ due to the curvedness of channel [33]. Here $\overline{U}$ and $\overline{W}$ are the velocity components along radial and axial directions respectively. Blood is considered as base fluid for both hybrid (Cu-Ag/blood) and two phase flow (Cu/blood) of nanofluid models.

The mathematical expression for cilia tips envelope is written as [34].

$$\overline{R} = a + a\varepsilon Cos\left(\frac{2\pi}{\lambda}\left(\overline{Z}-c\overline{t}\right)\right) = \overline{f}\left(\overline{Z},\overline{t}\right),$$

$$\overline{Z} = \overline{Z}_o + a\varepsilon\alpha Sin\left(\frac{2\pi}{\lambda}\left(\overline{Z}-c\overline{t}\right)\right) = \overline{g}\left(\overline{Z},\overline{Z}_o,\overline{t}\right), \tag{1}$$

The velocities of cilia tips can be represented as

$$\overline{W} = \left(\frac{\partial\overline{Z}}{\partial\overline{t}}\right)_{\overline{Z}_o} = \frac{\partial\overline{g}}{\partial\overline{t}} + \frac{\partial\overline{g}}{\partial\overline{Z}}\frac{\partial\overline{Z}}{\partial\overline{t}} = \frac{\partial\overline{g}}{\partial\overline{t}} + \frac{\partial\overline{g}}{\partial\overline{Z}}\overline{W},$$

$$\overline{U} = \left(\frac{\partial\overline{R}}{\partial\overline{t}}\right)_{\overline{Z}_o} = \frac{\partial\overline{f}}{\partial\overline{t}} + \frac{\partial\overline{f}}{\partial\overline{Z}}\frac{\partial\overline{Z}}{\partial\overline{t}} = \frac{\partial\overline{f}}{\partial\overline{t}} + \frac{\partial\overline{f}}{\partial\overline{Z}}\overline{W}, \tag{2}$$

By using Eq.(2) in Eq.(1), we have

$$\overline{W} = \frac{-\left(\frac{2\pi}{\lambda}\right)\left[\varepsilon\alpha ac Cos\left(\frac{2\pi}{\lambda}\left(\overline{Z}-c\overline{t}\right)\right)\right]}{\left[1-\left(\frac{2\pi}{\lambda}\right)\left\{\varepsilon\alpha a Cos\left(\frac{2\pi}{\lambda}\left(\overline{Z}-c\overline{t}\right)\right)\right\}\right]},$$

$$\overline{U} = \frac{\left(\frac{2\pi}{\lambda}\right)\left[\varepsilon ac Sin\left(\frac{2\pi}{\lambda}\left(\overline{Z}-c\overline{t}\right)\right)\right]}{\left[1-\left(\frac{2\pi}{\lambda}\right)\left\{\varepsilon\alpha a Cos\left(\frac{2\pi}{\lambda}\left(\overline{Z}-c\overline{t}\right)\right)\right\}\right]}, \tag{3}$$

We can distinguish between cilia's effective as well as recovery strokes by these two velocities that are given above. Here $\overline{W}$ as well as $\overline{U}$ represent the effective and recovery strokes respectively.

The dimensional form for governing equations take the form [35]

$$\frac{\partial}{\partial\overline{R}}\left(\left(\overline{R}+R^*\right)\overline{U}\right) + R^*\frac{\partial\overline{W}}{\partial\overline{Z}} = 0, \tag{4}$$

$$\rho_{hnf}\left(\frac{\partial\overline{U}}{\partial\overline{t}} + \overline{U}\frac{\partial\overline{U}}{\partial\overline{R}} + \frac{R^*\overline{W}}{\overline{R}+R^*}\frac{\partial\overline{U}}{\partial\overline{Z}} - \frac{\overline{W}^2}{\overline{R}+R^*}\right)$$
$$= -\frac{\partial\overline{P}}{\partial\overline{R}} + \mu_{hnf}\left[\frac{1}{\left(\overline{R}+R^*\right)}\frac{\partial}{\partial\overline{R}}\left\{\left(R^* + \overline{R}\right)\frac{\partial\overline{U}}{\partial\overline{R}}\right\} + \left(\frac{R^*}{R^*+\overline{R}}\right)^2\frac{\partial^2\overline{U}}{\partial\overline{Z}^2} - \frac{\overline{U}}{\left(R^*+\overline{R}\right)^2} - \frac{2R^*}{\left(R^*+\overline{R}\right)^2}\frac{\partial\overline{W}}{\partial\overline{Z}}\right], \tag{5}$$

$$\begin{aligned}\rho_{hnf}\left(\frac{\partial \overline{W}}{\partial \overline{t}}+\overline{U}\frac{\partial \overline{W}}{\partial \overline{R}}+\frac{R^{*}\overline{W}}{\overline{R}+R^{*}}\frac{\partial \overline{W}}{\partial \overline{Z}}+\frac{\overline{WU}}{\overline{R}+R^{*}}\right)\\ =-\left(\frac{R^{*}}{\overline{R}+R^{*}}\right)\frac{\partial \overline{P}}{\partial \overline{Z}}+\mu_{hnf}\left[\frac{1}{(\overline{R}+R^{*})}\frac{\partial}{\partial \overline{R}}\left\{\left(R^{*}\right.\right.\right.\\ \left.\left.+\overline{R}\right)\frac{\partial \overline{W}}{\partial \overline{R}}\right\}+\left(\frac{R^{*}}{R^{*}+\overline{R}}\right)^{2}\frac{\partial^{2}\overline{W}}{\partial \overline{Z}^{2}}-\frac{\overline{W}}{(R^{*}+\overline{R})^{2}}+\left.\frac{2R^{*}}{(R^{*}+\overline{R})^{2}}\frac{\partial \overline{U}}{\partial \overline{Z}}\right]+(\rho\alpha^{*})_{hnf}g(\overline{T}\\ -\overline{T}_{1}),\end{aligned} \tag{6}$$

$$(\rho C_{p})_{hnf}\left[\frac{\partial \overline{T}}{\partial \overline{t}}+\overline{U}\frac{\partial \overline{T}}{\partial \overline{R}}+\frac{R^{*}\overline{W}}{\overline{R}+R^{*}}\frac{\partial \overline{T}}{\partial \overline{Z}}\right]=k_{hnf}\left[\frac{\partial^{2}\overline{T}}{\partial \overline{R}^{2}}+\frac{1}{(\overline{R}+R^{*})}\frac{\partial \overline{T}}{\partial \overline{R}}+\left(\frac{R^{*}}{R^{*}+\overline{R}}\right)^{2}\frac{\partial^{2}\overline{T}}{\partial \overline{Z}^{2}}\right]+Q_{o}, \tag{7}$$

The flow is unsteady in fixed coordinates $(\overline{R},\overline{Z})$ but it turns out to be steady in a wave frame $(\overline{r},\overline{z})$. Since the flow and wave now moves with same speed in Z-direction. The transformations between fixed and moving frame are

$$\overline{z}=\overline{Z}-c\overline{t},\overline{r}=\overline{R},\overline{p}=\overline{P},\overline{w}=\overline{W}-c,\overline{u}=\overline{U}, \tag{8}$$

The dimensionless variables are

$$r=\frac{\overline{r}}{a},z=\frac{\overline{z}}{\lambda},w=\frac{\overline{w}}{c},u=\frac{\lambda\overline{u}}{ac},p=\frac{a^{2}\overline{p}}{c\lambda\mu},\beta=\frac{a}{\lambda},\theta=\frac{\overline{T}-\overline{T}_{1}}{\overline{T}_{0}-\overline{T}_{1}},s=\frac{R^{*}}{a},$$

$$\gamma=\frac{Q_{0}a^{2}}{(\overline{T}_{0}-\overline{T}_{1})k_{f}},G_{r}=\frac{(\rho\alpha^{*})_{f}ga^{2}(\overline{T}_{0}-\overline{T}_{1})}{c\mu_{f}},\epsilon=\frac{b}{a}, \tag{9}$$

Now using Eqs. (8) and (9) in Eqs. (4–7), then by using approximations $R_{e}\to 0$, $\lambda\to\infty$, we get the non-dimensional form of governing equations

$$\frac{\partial p}{\partial r}=0, \tag{10}$$

$$\begin{aligned}-\left(\frac{s}{r+s}\right)\frac{\partial p}{\partial z}+\frac{\mu_{hnf}}{\mu_{f}}\left[\left(\frac{1}{r+s}\right)\frac{\partial}{\partial r}\left\{\left(r+s\right)\frac{\partial w}{\partial r}\right\}-\frac{w+1}{(r+s)^{2}}\right]+\frac{(\rho\alpha^{*})_{hnf}}{(\rho\alpha^{*})_{f}}G_{r}\theta\\ =0,\end{aligned} \tag{11}$$

$$\frac{k_{hnf}}{k_{f}}\left(\frac{\partial^{2}\theta}{\partial r^{2}}+\frac{1}{r+s}\frac{\partial\theta}{\partial r}\right)+\gamma=0, \tag{12}$$

With non-dimensional boundary conditions

$$w=-1-\frac{2\pi\varepsilon\alpha\beta Cos(2\pi z)}{1-2\pi\varepsilon\alpha\beta Cos(2\pi z)},$$

$$atr=\frac{\overline{r}}{a}=\frac{\overline{h}(z)}{a}=h(z)=\frac{a}{a}+\frac{b}{a}Cos\frac{2\pi}{\lambda}(\overline{z})=1+\epsilon Cos(2\pi z),$$

$$\frac{\partial w}{\partial r}=0,atr=0, \tag{13}$$

$$\theta=0,\text{atr}=h(z)=1+\epsilon Cos(2\pi z),$$

$$\frac{\partial\theta}{\partial r}=0,atr=0, \tag{14}$$

Exact solution

The exact solution of Eqs. (11) and (12) satisfying the boundary conditions (13) and (14) are obtained directly by using Mathematica software and defined as

$$\begin{aligned}w(r,z)=&\frac{1}{180(\frac{k_{hnf}}{k_{f}})(-1+2\pi\varepsilon\alpha\beta Cos(2\pi z))(r+s)\left(h^{2}+2hs+2s^{2}\right)(\frac{\mu_{hnf}}{\mu_{f}})}\left[12G_{r}h^{5}(\right.\\ &-1+2\pi\varepsilon\alpha\beta Cos(2\pi z))\left(r^{2}+2rs+2s^{2}\right)\gamma(\frac{(\rho\alpha^{*})_{hnf}}{(\rho\alpha^{*})_{f}})-15G_{r}h^{4}(-1\\ &+2\pi\varepsilon\alpha\beta Cos(2\pi z))\left(r^{3}-r^{2}s-5rs^{2}-5s^{3}\right)\gamma(\frac{(\rho\alpha^{*})_{hnf}}{(\rho\alpha^{*})_{f}})-20G_{r}h^{3}(-1\\ &+2\pi\varepsilon\alpha\beta Cos(2\pi z))s\left(3r^{3}+5r^{2}s+rs^{2}+s^{3}\right)\gamma(\frac{(\rho\alpha^{*})_{hnf}}{(\rho\alpha^{*})_{f}})+2s\Big\{\\ &-45(\frac{k_{hnf}}{k_{f}})\frac{\partial p}{\partial z}(-1+2\pi\varepsilon\alpha\beta Cos(2\pi z))rs^{2}(r+2s)+90(\frac{k_{hnf}}{k_{f}})\left(qr^{2}+2s(r\right.\\ &\left.+s)\right)(\frac{\mu_{hnf}}{\mu_{f}})+G_{r}(-1+2\pi\varepsilon\alpha\beta Cos(2\pi z))rs(3r^{4}+15r^{3}s+55r^{2}s^{2}+75rs^{3}\\ &+30s^{4})\gamma(\frac{(\rho\alpha^{*})_{hnf}}{(\rho\alpha^{*})_{f}})\Big\}+h^{2}(-1+2\pi\varepsilon\alpha\beta Cos(2\pi z))\Big\{90(\frac{k_{hnf}}{k_{f}})\Big(\frac{\partial p}{\partial z}s^{3}-2(r\\ &+s)(\frac{\mu_{hnf}}{\mu_{f}})\Big)+G_{r}\left(3r^{5}+15r^{4}s-35r^{3}s^{2}-135r^{2}s^{3}-120rs^{4}\right.\\ &\left.-150s^{5}\right)\gamma(\frac{(\rho\alpha^{*})_{hnf}}{(\rho\alpha^{*})_{f}})\Big\}+2h\Big\{90(\frac{k_{hnf}}{k_{f}})\Big(\frac{\partial p}{\partial z}(-1+2\pi\varepsilon\alpha\beta Cos(2\pi z))s^{4}\\ &+\left(2\pi\varepsilon\alpha\beta Cos(2\pi z)r^{2}+2s(r+s)\right)(\frac{\mu_{hnf}}{\mu_{f}})\Big)+G_{r}(-1\\ &+2\pi\varepsilon\alpha\beta Cos(2\pi z))s\left(3r^{5}+15r^{4}s+25r^{3}s^{2}+15r^{2}s^{3}\right.\\ &\left.-30s^{5}\right)\gamma(\frac{(\rho\alpha^{*})_{hnf}}{(\rho\alpha^{*})_{f}})\Big\}+30(-1+2\pi\varepsilon\alpha\beta Cos(2\pi z))s\Big\{(h-r)s^{2}(h+r\\ &+2s)\left(3(\frac{k_{hnf}}{k_{f}})\frac{\partial p}{\partial z}-2G_{r}s^{2}\gamma(\frac{(\rho\alpha^{*})_{hnf}}{(\rho\alpha^{*})_{f}})\right)Log(s)+\Big(-3(\frac{k_{hnf}}{k_{f}})\frac{\partial p}{\partial z}(h+s)^{2}\left(r^{2}\right.\\ &\left.+2rs+2s^{2}\right)+G_{r}s\left(2s^{2}\left(r^{3}+2r^{2}s+rs^{2}+s^{3}\right)+h^{2}\left(r^{3}+3r^{2}s+3rs^{2}+3s^{3}\right)\right.\\ &\left.+2hs\left(r^{3}+3r^{2}s+3rs^{2}+3s^{3}\right)\right)\gamma(\frac{(\rho\alpha^{*})_{hnf}}{(\rho\alpha^{*})_{f}})\Big)Log(h+s)-(r+s)^{2}\left(h^{2}+2hs\right.\\ &\left.\left.+2s^{2}\right)\left(-3K\frac{\partial p}{\partial z}+G_{r}s(r+s)\gamma(\frac{(\rho\alpha^{*})_{hnf}}{(\rho\alpha^{*})_{f}})\right)Log(r+s)\Big\}\right],\end{aligned} \tag{15}$$

$$\theta(r,z)=\frac{\gamma[(h-r)(h+r+2s)+2s^{2}(-Log(h+s)+Log(r+s))]}{4K} \tag{16}$$

The volumetric flow rate defined by Eq. (17) and pressure gradient is calculated by using this volumetric flow rate

$$F=\int_{-h}^{h}wdr, \tag{17}$$

The dimensionless form for rate of fluid flow and pressure rise are computed by using following relations Fig. 1

$$Q=F+2,\Delta P=\int_{0}\frac{dp}{dz}dz, \tag{18}$$

Results and discussion

In this segment, the graphical results for the exact solutions that are obtained in previous section are discussed. The velocity profile $w(r,z)$ is plotted for different increasing values of physical parameters ε and s, as shown in Fig. 2a, (2b) respectively. Fig. 2a depicts that the velocity gains magnitude for both Cu/blood and Cu-Ag/blood, in the region $-1\leq r\leq 1.1$ as the values of ε increases but it shows completely opposite behaviour outside this region. Fig. 2b shows that the velocity increases for both Cu/blood and Cu-Ag/blood, in the region $-0.55\leq r$