Transp Porous Med (2017) 116:473–491
DOI 10.1007/s11242-016-0785-9

Triple-Diffusive Mixed Convection in a Porous Open Cavity

Mehdi Ghalambaz[1] · **Faramarz Moattar**[1] · **Abdolreza Karbassi**[2] · **Mikhail A. Sheremet**[3,4] · **Ioan Pop**[5]

Received: 9 April 2016 / Accepted: 13 October 2016 / Published online: 9 December 2016

Abstract The triple-diffusive mixed convection heat and mass transfer of a mixture is analyzed in an enclosure filled with a Darcy porous medium. The mass transfer buoyancy effects due to concentration gradients of the dispersed components (pollutant components) are taken into account using the Boussinesq approximation model. The governing equations are transformed into a non-dimensional form, and six groups of non-dimensional parameters, including Darcy–Rayleigh number, Peclet number, two Lewis numbers for pollutant components 1 and 2 and two buoyancy ratio parameters for pollutant components 1 and 2, are introduced. The governing equations are numerically solved for various combinations of non-dimensional parameters using the finite element method. The effect of each group of non-dimensional parameters on the pollutant distribution and the heat transfer in the cavity is discussed. The results indicate that the presence of one pollutant component can significantly affect the pollutant distribution of the other component. When the Lewis number of a pollutant component is small, the increase in the bouncy ratio parameter of the proposed component always increases the Nusselt and Sherwood numbers in the cavity.

Keywords Mass transfer · Three-component mixture · Pollutant diffusion · Environment

✉ Faramarz Moattar
namoattar@yahoo.com

[1] Department of Environmental Engineering, Ahvaz Branch, Islamic Azad University, Ahvaz, Iran

[2] Graduate Faculty of Environment, University of Tehran, P.O. Box 14155-6135, Tehran, Iran

[3] Department of Theoretical Mechanics, Faculty of Mechanics and Mathematics, Tomsk State University, Tomsk, Russia 634050

[4] Department of Nuclear and Thermal Power Plants, Tomsk Polytechnic University, Tomsk, Russia 634050

[5] Department of Mathematics, Babeş-Bolyai University, 400084 Cluj-Napoca, Romania

1 Introduction

The study of multi-diffusive natural convection in porous media is of interest in a wide range of industrial applications. For example, the migration of odor in a moisture air through fibrous of an insulation media is a triple-diffusive problem. When the variation of the concentration and the thermal expansion coefficient of the components in the mixture are significant, the buoyancy force due to mass transfer could play an important role on the flow circulation and distribution of components in the cavity. As the number of components increases, the number of buoyancy forces increases, and as a result, the behavior of the mixture gets more complex. For example, consider a problem in which a light gas leaks from a broken pipe in soil, saturated with a moisture air. The moisture of the air can significantly affect the dispersion behavior and the distribution of the gas in the soil. Therefore, modeling and understanding the behavior of a mixture is a very important problem in many of engineering applications, as it could provide a powerful mean for designing devices and structures for different possible goals such as separating the components of mixture, preventing dispersion of a contaminant (or a component) or enhancing the dispersion of a component in the mixture.

Different aspects of the double-diffusive heat and mass transfer in porous media have been studied in the literature. For instance, Chamkha and Al-Naser (2001) have considered the double-diffusive convective heat transfer of a mixture in a cavity with opposing temperature and concentration gradients. In the study of Chamkha and Al-Naser, the buoyancy force due to concentration gradient is opposed to the buoyancy force due to thermal expansion of the fluid. The authors have also studied the effect of cavity inclination angle. Following the study of Chamkha and Al-Naser (2001) and Chamkha (2002) has examined the effect of adding concentration of the flow heat and mass transfer of a binary fluid in an enclosure. Later, Chamkha and Abdulgafoor (2006) have addressed the influence of non-Darian effects of the double-diffusive heat and mass transfer of a binary fluid in an enclosure. Bourich et al. (2004) have studied the natural convection heat and mass transfer in a cavity heated from bottom and cooled from top while differentially salted from side walls. Kuznetsov and Sheremet (2009) have analyzed double-diffusive natural convective in a rectangular cavity bounded by solid heat-conducting walls with local heat and contaminant sources. Later, Kuznetsov and Sheremet (2011) have extended their previous study to the case of a three-dimensional cavity with solid walls and local heat and contaminant sources.

In the past years, the mixed convection heat and mass transfer of binary fluids has been examined by many researches; for example, Mahmud and Pop (2006) have studied the effect of buoyancy forces due to concentration gradients on the mixed convection in a cavity. Later, Chamkha et al. (2012) and Ravikumar et al. (2013) have addressed the hydromagnetic and chemical reactive effects on the double-diffusive mixed convective heat and mass transfer of a binary fluid in a cavity. Sheremet et al. (2015) have extended the problem of double-diffusive mixed convection in a cavity to the case of nanofluids.

As seen, the review of the literature indicates that the double-diffusive heat and mass transfer in enclosure has been addressed in many of previous researches; however, there are only few researches which have considered triple- or multi-diffusive heat and mass transfer of a mixture.

Rionero (2012, 2013, 2014, 2015) has studied the onset of instability of a three-component mixture in porous media. The results indicate that the mass diffusion is an important effect for the onset of convection. There are also few studies on triple-diffusive convection of a mixture in boundary layer.

Khan et al. (2013), Khan and Pop (2013) and Khan et al. (2015) have addressed the triple-diffusive convective heat and mass transfer of a mixture in the boundary layer over a flat plate

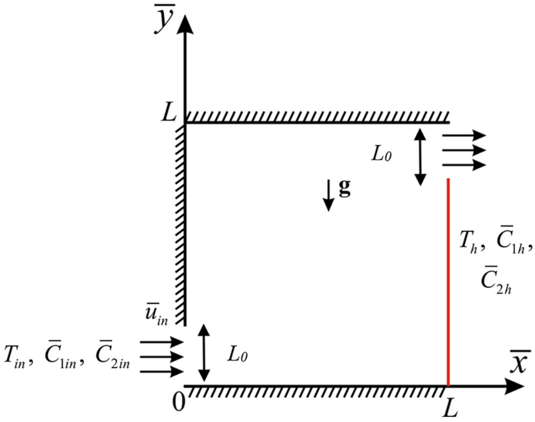

Fig. 1 Schematic diagram of coordinate system and physical model

placed in a saturated porous medium. These authors (Khan et al. 2013; Khan and Pop 2013; Khan et al. 2015 have utilized the boundary layer approximations and then transformed the governing equations into ordinary differential equations using similarity method. The authors have concluded that the effect of buoyancy forces due to concentration gradient is important in the boundary layer.

Very recently, Ghalambaz et al. (2016) have examined the triple-diffusive natural convective heat transfer of a three-component solution in a cavity using the finite element method. Following the study of Ghalambaz et al. (2016), the present study aims to address the triple-diffusive mixed convection of a three-component solution in an enclosure filled with a porous medium.

2 Basic Equations

We consider the steady triple-diffusive convective flow in an open cavity filled with a fluid-saturated porous medium. A schematic geometry of the problem under investigation is shown in Fig. 1, where $\bar{x}$ and $\bar{y}$ are the Cartesian coordinates measured along the lower wall and along the vertical wall of the cavity. The flow in the cavity is considered to be steady, laminar and incompressible. The domain of interest is a square cavity of size L with an inlet of size L_0 in the bottom part of the left vertical wall and an outlet of the same size in the upper part of the right vertical wall. The right vertical wall of the cavity is maintained at the constant temperature T_{h} and concentrations $C_{1\mathrm{h}}$ and $C_{2\mathrm{h}}$. The rest of the walls of the cavity are assumed to be adiabatic and impermeable. In addition, the incoming flow through the inlet is at a uniform horizontal velocity $\bar{u}_{\mathrm{in}}$ ($\bar{v}_{\mathrm{in}} = 0$) at the constant ambient temperature T_{in} and concentrations $C_{1\mathrm{in}}$ and $C_{2\mathrm{in}}$ where $T_{\mathrm{h}} > T_{\mathrm{in}}$, concentrations $C_{1\mathrm{h}} > C_{2\mathrm{in}}$ and $C_{2\mathrm{h}} > C_{2\mathrm{in}}$.

Following Rionero (2013), we also assume that two different chemical components ("pollutants") S_m $(m = 1, 2)$ have dissolved in the fluid-saturated porous medium, which have concentrations C_m $(m = 1, 2)$, respectively, and that the equation of state is

$$\rho = \rho_0 \left[1 - \beta_T (T - T_{\mathrm{in}}) - \beta_1 (C_1 - C_{1\mathrm{in}}) - \beta_2 (C_2 - C_{2\mathrm{in}})\right], \tag{1}$$

where ρ_0 is a reference density and the constants β_T, β_1 and β_2 denote the coefficient of thermal expansion and solute S_m expansion coefficients, respectively $(m = 1, 2)$, which are defined by